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BUSINESS STATISTICS
AND
STATISTICAL METHOD

BY

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PREFACE

The aim of this book is to provide a simple but comprehensive course in Business Statistics, and to present the subject in such a way that those using the book may acquire an adequate knowledge of (*a*) the procedure for the proper compilation, presentation and interpretation of statistical information, and (*b*) the usefulness and importance of statistical method and analysis as an aid to the successful conduct of business.

The training of those who are to become the business men and women of the future should include a course in Business Statistics, so that when responsibilities are assumed they may confidently attempt the analyses of business problems, and be able to exercise more efficient control. The control of a business is facilitated by an adequate review of the facts obtainable by the application of practical statistical methods, particularly when used in conjunction with reliable costing methods. A training in statistical method is necessary also for those who are to become members of the clerical staff responsible for the compilation, analysis, and presentation of facts for the information of their executive officers.

The importance of the study of Business Statistics is sufficiently indicated by the fact that the subject is included in many schemes of study for the examinations for the Board of Education Endorsed Certificates in Commerce. In the opinion of the author, the value of these certificates would be enhanced if the subject were made compulsory as a second- or third-year subject.

The scope of the book is sufficient not only for the requirements of these examinations, but also for those of the London Chamber of Commerce, the Royal Society of Arts, and the Union of Lancashire and Cheshire Institutes.

As regards the present treatment of the subject, it has been thought desirable to describe first the technique and principles of statistical method in so far as these may be in any way applicable to business data, and then to supplement the description by illustrations (coupled with suggestions) of their practical application and interpretation. Unfortunately it has been necessary to give a fuller treatment of statistical method and theory than would appear to be necessary for ordinary business purposes, owing to the fact that many questions included in examinations in Business Statistics indicate that a very wide view of the subject is taken by the examiners.

Still, an endeavour has been made to make the book sufficiently practical for the application of the fundamental principles of statistics to actual business needs, and no attempt has been made to go further than this. It is hoped therefore that the book may assist, in some small measure, to the more general use of statistical methods in business.

H. J. W.

September 10, 1936.

PREFACE TO SECOND EDITION

In this new edition I have taken the opportunity to make a number of additions to the text and to effect corrections.

I desire also to extend sincere thanks for the many expressions of appreciation sent to me by teachers, students and business men.

H. J. W.

October 15, 1940.

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CHAPTER 1

INTRODUCTION

THERE are many factors which influence the success or failure of any business, and of these some are external and beyond the business man's control, but others are connected with internal conditions which can be definitely controlled by the application of sound management through the exercise of sound judgments based upon ascertained facts. In manufacturing businesses it is now very generally recognised that a suitable system of Costing and Cost Accounting is essential to guide the manufacturer in the control of his affairs. In all businesses, however, statistics are equally necessary, for it is indisputable that many facts are obtainable only by statistical analyses, enabling accurate information to be presented in an appropriate manner.

Ability to employ the technique of statistical methods and to interpret the significance of statistical reports is necessary for the application of that skill and precision so vitally important to cope with the increasing complexity and vigour of competition which exists to-day in all spheres of business.

The Value of Business Statistics may be broadly summarised as follows :—

1. By the adoption of suitable statistical methods, determinable facts can be weighed in the balance to assist in judging possibilities and probabilities from ascertained results and tendencies.

2. Training in the subject provides ability not only to prepare statistical information, but also to interpret it with a view to any necessary action.

3. Statistical analyses are of great practical value in almost every class of undertaking, for assisting in the formulation of policy, and often for the forecasting of business conditions of the immediate, and sometimes the more distant future.

4. Analysis and comparison of past facts can be used to reveal what changes have occurred, and data relating to current facts may indicate tendencies which it is necessary to retard or develop.

5. A knowledge of statistical method gives ability to undertake market research, interpret statistics of prices and other data concerning industry and commerce in general, and to obtain thereby further information towards the formulation of business policy.

The Definition of Statistics.—Statistics is a science devoted to the measurement and analysis of facts numerically, and to their presentation in a manner suitable to permit the ascertainment of the relationships between those facts. Business Statistics is the application of statistical methods with special reference to the needs of those in industry and commerce in all its forms.

Professor A. L. Bowley, in his *Elements of Statistics*, defines the subject as "the science of measurement of the social organism regarded as a whole, in all its manifestations," but he also refers to it as "the science of averages" and "the science of counting." Another definition given by W. L. King, in his *Elements of Statistical Methods*, states that "the science of statistics is the method of judging collective natural or social phenomena from the results obtained by the analysis of an enumeration, or collection of estimates."

The Main Divisions of the Study of Statistics may be regarded as follows :—

1. *Statistical Method*, which formulates rules of procedure and general principles applicable to all kinds or groups of data.
2. *Applied Statistics*, which deals with the application of statistical methods and rules to concrete facts or subject matter.

Applied Statistics may be either

(*a*) *Descriptive*, as when dealing with known data or records relating to the past, or present. "Business Statistics" comes within this division.

(*b*) *Scientific*, as when used for the determining of physical and psychological laws by the investigation of data collected for descriptive purposes by statistical methods.

Applied statistics, both descriptive and scientific, is the division of the subject used by economists, sociologists, biologists, and other investigators to demonstrate or establish scientific laws and theories.

Business Statistics is a branch of descriptive applied statistics which deals with the analysis, measurement, and presentation of business facts. Business Statistics comes under the heading "descriptive" inasmuch as the data are collected, analysed, and presented, usually in tabulations or graphical form, to provide information upon which executive judgments may be formed.

The study of statistics must include the consideration of methods suitable for application in the field of business activities. In fact Business Statistics is, as previously stated, the special application of statistical methods to the activities of business enterprise. Statistical Method.

Statistical Method.—The collection, analysis, and presentation of data, in a manner which will give desired information, necessitates the use of statistical method, which provides certain rules of procedure and principles essential for the requisite degree of accuracy.

For practical considerations, Business Statistics are generally dealt with on a broader and simpler basis than is required for academic statistical investigations, but notwithstanding this the student preparing for examinations in the subject is required to be conversant with the main principles of statistical method.

The scope of Statistical Method to be considered includes :—

(*a*) Rules for the collection of data for the presentation of tabular, graphic and other reports and statements.

(*b*) Procedure for the questionnaire method of primary investigation.

(*c*) The compilation of index numbers.

(*d*) The procedure for ascertaining suitable averages by different methods.

(*e*) Simple treatment of the measurement of variation (dispersion) as shown by the deviation of items in a group or series from types ascertained in various ways.

(*f*) Consideration of the extent to which two or more groups of data under comparison are related. This relationship is referred to as correlation.

(*g*) The study of seasonal variations, business cycles, various index numbers, basic trend in business and industry, business forecasts and budgets, including budgetary control.

(*h*) Modes of graphic presentation of facts.

(*i*) Interpretation of published statistics and their use as sources of information.

(*j*) The sources of information for business statistical purposes.

Reference may be made here to the fact that very frequently figures are produced which are misleading or fallacious. Sometimes this is due to ignorance of statistical methods, and sometimes to unscrupulousness. There is a difference between statistics and mere figures. Figures compiled must be relevant to the purpose in view and there must be proper regard to statistical requirements for accuracy.

It is not true that "statistics can be made to prove anything." This is true only of figures which do not comply with statistical requirements. Fallacious arguments may be based on figures and information which are incomplete or not representative, but such figures are not statistics. Figures compiled having proper regard to the principles comprised in statistical method cannot support fallacious contentions so long as they are used for purposes for which they were compiled, or incidental thereto.

QUESTIONS

1. "Statistics can be made to prove anything." Comment on this statement.

2. "Figures cannot lie." Discuss this argument and explain the difference between figures and statistics.

3. In what practical ways can statistics be of service to a business man?

4. "Statistics is the science of counting." Criticise this definition. If you are not satisfied with it, state your own. (*Incorporated Accountants* (*Final*).)

5. Classify the different branches of statistics, and describe each briefly. (*Incorporated Accountants* (*Final*).)

CHAPTER II

PRELIMINARIES TO THE COMPILATION OF DATA

Requirements.—An all-important step in statistical work is the collection of figures and facts, involving consideration as to how these may be obtained, and as to the most suitable mode of presentation.

Although this collection of data is commonly regarded as simple, it frequently is not, and in any case requires considerable thought, inasmuch as the details collected form the foundation of the statistical information to be provided.

Every feature of the problem or inquiry must be examined, so that the purpose of the statistics may be properly fulfilled.

Three aspects to be borne in mind are :

(1) What information is desired ?
(2) By whom is it desired ?
(3) For what purpose ?

The steps to be taken are therefore (*a*) to define the problem ; (*b*) to determine the available sources of information ; (*c*) to define the statistical units which are to be dealt with ; (*d*) to decide upon the mode of treatment, having regard to the purpose for which the statistics are required.

These matters will now be considered in turn.

Defining the Problem.—The importance of a clear and concise statement of the information required cannot be over-estimated. A clear definition of the problem not only tends to closer accuracy, but also saves time in the collection and arrangement of data. When the problem has been so defined, all relevant elements must be sought for inclusion in the inquiry, whilst the omission of facts can be guarded against, as can the admission of unnecessary data.

In business the information required by the executive may necessitate the resolving of the problem into two or more distinct statistical problems involving different methods of determination.

This often occurs, for example, in reference to information required concerning factory labour and wages: Is it wages earned or wages rates about which data are required? Must the statistics concern all employees, or separately, grades, and men and women? Should lost time, overtime, piece-work, and bonus payments be included or allowed for? Should receipts in kind be included? Consider, for example, the particulars compiled in the statistical report given in Fig. 5.

The problem having been defined, the investigation can be planned.

Planning the Inquiry.

1. *Purpose.*—When planning, it is desirable to know the purpose of the investigation. This knowledge serves as a guide during the course of collection of information, and enables the compiler to decide questions and difficulties which may arise in the process.

The purpose being kept in mind ensures uniformity in the disposition of difficult items.

The purpose may be general or specific. Thus in a business there may be a general scheme for recording comprehensive statistics for basic records, but more usually there is a specific objective, and only those items concerning the specific end in view are collected, as, for instance, the percentage efficiency of piece-workers in a factory.

Examples of statistics compiled for general purposes are the Census of Population, Census of Production, Board of Trade Returns of Imports and Exports, etc.

2. *Scope.*—The collection of data in business for the general purpose of basic records must obviously be comprehensive, with the danger, if not limited in scope, of the accumulation of statistics that have little or no value. It is therefore desirable that statistical compilations should be only for such specific purposes as will assist the management in its control of definite aspects of the organisation. This is a matter which requires careful consideration, as no compilation should be undertaken unless of actual practical service to the management. Further, the cost of compilation must be weighed against the value of the information to be afforded. The principle involved is the same as that applicable to any service or production department—namely, the provision of only such service or production as can be profitably utilised.

It follows that before the work is commenced the extent of the data required should be determined. For some purposes full records may be unnecessary, and the desired information can be obtained from a review of selected representative facts, *i.e.* by the method known as "sampling," which will be discussed in a later chapter.

Degree of Accuracy.—Complete accuracy may be attainable, and is required, in many cases, but not in others. For many statistical inquiries a reasonable degree of accuracy is all that is necessary or obtainable. Having regard to the purposes of the compilation, a decision must be made as to the degree of accuracy desired, and the plans for collecting the data arranged accordingly. The standard of accuracy should be stated, indicating if possible the limits of probable error.

Sometimes an approximately accurate report presented speedily is more valuable than a precise one delayed.

Statistical Units.—The compilation of statistics necessitates counting or measurement, hence it is essential to define the unit or units to avoid the danger of fallacious results through the inclusion of items which should be omitted, and the omission of items which should be included. It is self-evident that we should know what is to be counted or measured before beginning the collection of the data, hence the vital importance of defining the statistical unit which is to form the basis of any group of data.

The definition of the unit is not always as simple as would at first appear to be the case. The meaning of a word commonly understood in conversation or in ordinary writing is frequently too wide, and for statistical purposes a restricted specification or unmistakable definition must be applied. A little reflection on the following words will reveal the need for precise specification in each case; prices may be retail, wholesale or cost; accident may refer to a slight or serious injury, one officially reported, or one resulting in a compensation claim; wages may be the amount earned by workers directly engaged in production, or by all labour in the workshops, or by all in the business; it may include overtime and bonus payments.

When considering published statistics on trade, shipping, unemployment, the exact interpretation of them necessitates a clear conception of the units dealt with; some observations on these are given in a later chapter dealing with published statistics.

Not only must the unit be unmistakably defined, but it must also be clearly understood by all taking part in the enumeration and compilation of the data.

The Requirements for a Statistical Unit may be summarised thus:

(1) *It must be Specific and Unmistakable.*—If other meanings than that adopted apply to the term used, instructions as to method of treatment in the compilation must be stated.

(2) *It Must be Homogeneous.* This uniformity is essential. The unit must not imply different features or characteristics at different times and places. If the selected unit is not applicable to all the cases coming under review, it is often possible to overcome the difficulty (*a*) by sub-dividing the data into groups or classes until sufficient uniformity has been secured, (*b*) by expressing dissimilar units in terms of equivalents of the selected unit. *E.g.* if the output of bakeries for a period were being compared, some producing 1, 2 and 4 lb. loaves, all sizes could be reduced to the equivalent of 2 lb. loaves, which would probably be sufficient for the purposes of the comparison.

(3) *It Should be Stable.*—When it is desired to use a fluctuating unit such as the dollar or £ sterling, the value on a particular date may be selected as the standard unit into which current values may be converted by the use of suitable coefficients. Thus if the £ this year will buy only as much as 15s. at a standard selected date, the coefficient applicable is 0·75, so that £100 this year represents £75 as measured by the standard £ sterling unit.

(4) *It Must be Appropriate to the Inquiry and be Capable of Correct Ascertainment.*—The number of employees on the staff of a factory from the production engineer's point of view is number engaged in his workshops, but from the standpoint of the general manager it is the production staff, augmented by the employees in the service departments and offices plus those of the sales department. When compiling labour statistics, it is necessary to select the unit appropriate to the information required, *e.g.* workers engaged directly on production, those employed in indirect factory services, those in the administrative offices or in the sales department.

Types of Statistical Units.—*A Simple Statistical Unit*, such as many in common use, *e.g.* yards, pounds weight, bushels, hours, reams, £ sterling, marks, lire, etc., are not difficult to define. Care has to be taken in the actual use of some of them. For example, a ream may be 500 or 512 sheets, a ton may refer to a long ton or 2240 lbs., a metric ton of 2204·6 lbs., in shipping a measurement ton of 40 cubic feet, gross tonnage representing the cubic feet capacity of a whole vessel divided by 100, and so on.

Again, monetary units vary from time to time, and the same unit may have a different value in different countries; hence allowance for such variations must be considered. A working day may be one of say 10, 8 or 6 hours, according to the industry; hence statistics concerning working days must be considered accordingly, and the day defined.

A Composite Unit may have to be used in some cases. Thus electric power may be measured in units of kilowatt-hours; railway transport in ton-miles (*i.e.* number of tons × number of miles carried), omnibus transport in bus-miles; labour force in direct-labour hours; velocity in feet per second or miles per hour; pressure in foot pounds per minute and so on.

Classification of Statistical Data. Data are usually classified on the basis of proximity to original observations into two classes: (*a*) Primary, (*b*) Secondary.

Primary Data are data gathered by original observation, measurement, count, or original recordings.

Secondary Data are those employed which have been gathered originally by someone else. For example, data collected for the official Census of Production are primary data for the Census officer, but are secondary data when used by others.

Figures collected in the actual field of enumeration or observation are primary data, but figures taken from reports prepared by someone else are secondary data.

QUESTIONS

1. Indicate the main uses of Statistics. (*Incorporated Accountants (Final).*)

2. Illustrate the difficulties of determining the statistical unit. What are its necessary characteristics? (*Incorporated Accountants (Final).*)

3. Discuss the importance of exact definition in statistics. Illustrate your answer by reference to either (*a*) occupational statistics; or (*b*) statistics of the Balance of Trade. (*London Chamber of Commerce.*)

4. Why is statistical science necessary, and what are its uses? (*Incorporated Accountants (Final).*)

5. If you were invited to inquire into the wages paid in any industry, on what points would it be essential to be clear before you commenced work. (*Incorporated Accountants (Final).*)

CHAPTER III

SOURCES OF DATA

I. For General Statistical Purposes

Methods of Obtaining Primary Data may be considered under three headings :—

1. *Personal Collection.*

(*a*) For general statistical purposes the method of personal observation and collection of data may be used for localised inquiries on any particular subject.

(*b*) For business purposes all the internal records of production, labour time and wages, machine operating and idle time, stores receipts and issues, details of cost, administrative and sales records compiled internally are primary data, consideration of which will be dealt with later.

(*c*) Personal interviews to obtain answers to selected questions may be resorted to when available informers are reluctant or indifferent, when the investigation is one of complexity or when questions need explanations.

2. *By Scheduled Questionnaires.*

(*a*) Delivered and collected by canvassers or enumerators who may help the recipient to fill in the answers. This method usually produces good results (see (*c*) above).

(*b*) Posted to informants who are asked to return the forms by post. It has the disadvantage that in a public distribution many do not trouble to return the forms. The method may usually be used successfully for many business and industrial inquiries.

3. *By Local Reports* based on the observations of selected correspondents or local agents. Business representatives can help considerably for certain types of inquiry. Marketing data are frequently collected in this way. The data thus collected may be incomplete or biassed, but for some types of inquiry the method is sufficiently reliable,

and less expensive than the procedure of sending direct questionnaires.

Sources of Secondary Data.—(1) *Internal* reports and data may be prepared from internal reports and statements compiled from primary data; from reports from other business concerns; or from published business reports. The financial accounts and statements, usually prepared with great accuracy by the accounting department, and records and accounts compiled by the production and costing accounts, may be utilised by the statistical department for preparing administrative and production statistics.

(2) *External.*—For general statistical purposes much useful data collected by others, usually for other purposes, may be utilised as secondary data.

In connection with such information it is necessary to know:

(*a*) How far the data of the compiler are reliable, and whether he had the ability to obtain the true facts.
(*b*) The degree of accuracy of such data.
(*c*) From what source or sources were the data obtained.
(*d*) For what object was the primary compilation made.
(*e*) By what method the data were collected.

External secondary data may be obtained from :—

(i) *Private reports* from business and other agencies specialising in certain directions, Trade Associations and sometimes Chambers of Commerce.

(ii) *Published Information,** as for example:
  (*a*) Official publications by Government Departments, Municipalities, The League of Nations, Royal Commissions, Foreign Governments, etc.
  (*b*) Trade Associations, Chambers of Commerce, Banks, Societies, Stock Exchanges, etc.
  (*c*) Technical and trade journals, books and newspapers.
  (*d*) Reports by economists, statisticians, research agencies, University bureaux, educational associations.

Scrutiny of Primary Data.—Business data obtained internally should be, and usually are, subjected to some form of check whereby errors, omissions and defective reports may be remedied.

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* Described in Chapter XX.

Externally collected data are less easily verified, but all should be edited to discover inconsistencies and probable errors and omissions. Incomplete or defective schedules may have to be returned, unless it is possible to insert additions or corrections with reasonable safety. Manifestly erroneous figures and very unsatisfactory schedules should be ignored. In nearly all cases it is better to have a smaller number of correct schedules or data than to have a large number which includes many incorrect details. In the first case mathematical approximation may provide some remedy, but in the second case this is impossible.

Scrutiny of Secondary Data.—In addition to the inquiries referred to above in connection with primary data, care must be taken to see that terms used and the units employed correspond with those the investigator is using. A slight variation in definition or terminology may lead to serious discrepancy.

Continuous reports on a matter, particularly those covering a number of years, may not be strictly comparable owing to minor adjustments or changes in point of view of the compilers.

The comprehensiveness of the data should be particularly scrutinised. Thus unemployment statistics may refer to: (*a*) all unemployed; (*b*) unemployed persons who are members of trade unions; or (*c*) only those who are State insured, as in the case of the Ministry of Labour's figures.

In general, it must be emphasised that the original purpose for which data have been collected must be borne in mind and carefully considered before they are used for another statistical purpose. The suitability, adequacy and reliability of data for the purpose in view must be established by careful examination, particularly when obtained from external sources.

Necessary Attributes of Data.—A review of what is implied in the above remarks will assist in showing their importance:—

1. *Reliability.*

(*a*) Are both the source and original compilers trustworthy?

(*b*) Were proper methods and safeguards applied in the collection of facts?

(*c*) May there have been deliberate or unconscious bias on the part of the compiler?

(*d*) When were the data obtained and can that time be regarded as normal?

(*e*) What degree of accuracy was aimed at and to what extent was it secured?

The degree of accuracy although reliable enough for the original inquiry, may not be accurate enough for another purpose.

2. *Suitability* for the purpose in view has to be considered apart from reliability, as may be seen from the following examples:

(*a*) Prices may refer to a class of commodity, *e.g.* per ton of paper, irrespective of the weight per ream, which varies with the quality and thickness of the paper; steel, wool, cotton or grain prices may include various qualities, or be based on an average of different markets, instead of being for particular qualities or localities.

(*b*) Wages may refer to all grades, or to men and women, instead of to a desired grade, sex or occupation.

(*c*) Sales may represent orders received, orders despatched or orders charged and paid for, and may include either, or both, wholesale and retail sales.

3. *Adequacy.* Although reliable and suitable in character data may be inadequate for another purpose.

(*a*) Data may refer to a more limited or too extensive an area.

(*b*) It may not cover suitable periods, *e.g.* yearly instead of monthly.

(*c*) The degree of accuracy given, *e.g.* to the nearest 1000, although sufficient for the original purpose, may be too wide for the purpose in mind.

II. SOURCES OF BUSINESS DATA

Attention may now be directed more particularly to the sources of statistical data in a modern business concern.

There is no doubt that comparatively few business concerns have adequate statistical information, notwithstanding that a considerable amount of important primary data are regularly recorded for other purposes. It is true that many businesses conducted on modern lines are an exception to this general statement. Further, the developments in commercial education are helping increasingly towards a more enlightened body of business executives, who are able to realise the value of, and make use of, the valuable information afforded by an appropriate system of costing augmented by statistical reports.

Primary Business Records. Many records of original entry prepared for purposes of accounting and permanent records contain much detail, and, being based on proved primary data, they are the most valuable sources of information with which the statistical department can work. The records and statements regularly prepared by the Accounting and Costing Departments may be regarded as highly satisfactory secondary data upon which the statistical department can work, because as a rule they have been carefully prepared and checked against key figures or controls of the accounting departments.

Special Primary Records may also be required involving the collection of further new data and facts; but even for these it may often be arranged for the provision of the necessary data by an elaboration of certain details at the time of preparation of the usual reports and accounting records. If this is done, the expense and time attached to direct special collection by the statistical department are avoided.

Classification of Business Statistics.—It may be assumed that the management of all business organisations require statistical information for adequate control. The following is a broad classification of the information which normally will be advantageous:

1. Statistical statements or reports concerning the purchases of materials and expenditure on services and business expenses. Purchase control is an important matter. Internal Index Numbers showing the trend of prices and of production are invaluable.

2. In many businesses, data concerning production quantities, costs and efficiencies.

3. Statistics relating to employees, their occupations, earnings, service, output, efficiencies and conditions. In large businesses the extent and effect of labour turnover on costs have been revealed by statistics.

4. Statistics concerning marketing, sales and distribution, and the relationship of stocks for sale. Various methods of indicating trend, not only of prices, but also of public taste can be presented.

5. Statistical compilations facilitate budgeting and budgetary control, with the advantages of estimating in various directions for the immediate and distant future.

6. Information concerning capital, its acquisition, dis-

position, use and the return upon it. Statistics relating to fixed capital in the form of plant, etc., and of floating capital, particularly that locked up in stores materials and finished stocks, must be reviewed frequently having regard to the extent of liquid resources.

In presenting such information outlined above, the method of presentation, whether in tabular, graphic or diagrammatic form, is a matter which demands a knowledge of statistical method, and the ability to select the most advantageous manner of conveying the desired information to the executives.

In large establishments the scope is, of course, greater than in small businesses, but in every case some knowledge of recognised statistical methods is an advantage.

QUESTIONS

1. What main factors must be borne in mind when compiling data ?
2. What are the requirements for a good statistical unit ?
3. Distinguish between primary and secondary data, and state to what extent they may be considered reliable.
4. What information would you require to satisfy yourself as to the reliability of secondary data ?
5. From what sources (*a*) internal, (*b*) external, can useful statistical data be obtained by the statistical department of a business ?
6. Name four ways in which statistics may be of service to a manufacturer.
7. Write notes upon the questionnaire method of primary investigation. (*London Chamber of Commerce.*)
8. Discuss the importance of exact definition in statistics. Illustrate your answer by reference to either (*a*) occupational statistics, (*b*) statistics of the Balance of Trade. (*London Chamber of Commerce.*)
9. Describe sources of statistical information on the conditions of any industry known to you. (*London Chamber of Commerce.*)
10. Why are schedules in charge of enumerators the best method of conducting extensive statistical inquiries ? (*Incorporated Accountants* (*Final*).)
11. What are the advantages and disadvantages of collecting statistical data by means of schedules to be filled in by informants ? (*Incorporated Accountants* (*Final*).)
12. Classify the methods of statistical investigation, and state briefly their respective merits and demerits. (*Incorporated Accountants* (*Final*).)

CHAPTER IV

STATISTICAL SAMPLING

Representative Items or Samples.—For many purposes approximate and sufficiently accurate information concerning a mass or group of items can be obtained by selecting consciously, or at random, a number of representative items or samples, which when reviewed and interpreted enable approximate conclusions to be established concerning the whole group. Random selection, when made of a reasonably large number of items, gives a sufficiently reliable picture or indication of the whole group of which the samples are a part. The method saves the trouble and expense of complete enumeration and analysis, and for many purposes leads to surprisingly accurate conclusions.

The " Population " or " Universe."—The group of data from which samples are taken is termed the " population " or " universe." For example, if the output of, say, 100 pieceworkers producing a certain article be selected at random from, say, 1000 workers, these outputs will be reasonably representative on the average, of the output of all the workers. The population or universe is the entire group of 1000 output quantities.

Not only is this an economical method, but, at times, is the only method of obtaining certain kinds of information, particularly when the " population " of the items is very extensive.

Two Basic Laws.—This method of statistical induction is based on two general laws : (*a*) The Law of Statistical Regularity; (*b*) The Law of Inertia of Large Numbers. These laws or principles are an important feature in much statistical work, particularly in relation to sampling, and should be fully understood.

Law of Statistical Regularity.—This states that a reasonably large number of items selected at random from a large group of items, will, on the average, be representative of the characteristics of the large group (or " population ").

The important features are :—

(*a*) The selection of samples must be made at random, as for instance, by some lottery method, so that every item in the "population" has a chance of being a sample.

(*b*) The number of samples must be large enough to avoid the undue influence on the average of abnormal items. The larger the number of items selected the more reliable is the information afforded. It can be shown from statistical calculations that the reliability of the indication of the sampling is proportional to the square root of the number of items included. (See page 138.)

The principle may be illustrated by some simple instances.

(1) Assume a sack contains 100,000 discs red on one side, black on the other. If successive lots of 100 are taken out and dropped haphazard, the numbers of red surfaces and of black ones will be approximately equal, and the greater the number of lots so dropped the more close this equality is likely to be.

(2) If the percentage return on invested capital of a considerable number of drapery stores throughout the country can be ascertained from published and private reports as being 7½ per cent. on the average, in all probability it would be found that the percentage calculated from a summary of the return shown by every store in the country, if obtainable, would approximate to 7½ per cent. By recording known results of the "sample" stores annually, the trend of any rise or fall of the percentage return might reasonably be accepted as the likely trend of profit fluctuation for the drapery store trade as a whole.

(3) A collection of variously coloured and designed boxes containing stationery shown as an advance display in various trade centres throughout the country revealed that six of the designs were selected by 80 per cent. of the buyers viewing the collection, and by finding the average percentage quantity ordered of each design the manufacturer was able to plan his production output. The ultimate sales proportions came very close to the estimated ones based on "sampling" calculations.

Law of Inertia of Large Numbers.—This law is a corollary of the law just described. It states that large groups or aggregates of data show a higher degree of stability than small ones.

The movements of all the separate components of the aggregate reveal a tendency to compensate one another, some probably moving higher, others lower, but taking a large number of data, it is unlikely that they will all move in the same direction. The greater the number composing the aggregate, the greater will be the compensation, or tendency of movements to neutralise one another, and consequently the more stable will be the aggregate.

The law does not mean that changes cannot take place over a period. It means that when very large numbers are involved changes are likely to be more gradual and regular than when relatively small numbers or quantities are considered.

Secular changes arising from some fundamentally changing conditions due to cumulative long-period tendencies are not precluded by the law of inertia of large numbers. New laws of government, and scientific developments affect the death rate, and many superimposed modern conditions affect the birth rate. Road deaths and accidents increased with the modern developments in road transport, but are being retarded by stringent regulations. Production in factories has been constantly changing owing to inventions and discoveries, whilst factory regulations have introduced other influential factors. But irrespective of such general movements or trends, the law still applies.

A good example is that of crops and production of certain commodities. Taking figures for the whole world, the total remains fairly constant for these commodities, and although over a long series of years they may fluctuate, the fluctuations are far less than those based on figures for individual countries and areas.

Sampling by Random Selection of items from a large " population " depends upon the theory of probability, out of which the two laws just described have emerged.

Sampling by Conscious Selection is less dependent upon probabilities, and in business statistics may often be used with satisfactory results. In some cases there is greater accuracy than when random selection has to be relied upon, but biassed choice is a danger.

Sampling by Random Selection.—The methods of choosing samples are various, *e.g.* by making a draw as in a lottery; taking every tenth example (or other interval according to the nature and quantity of the " population "); pricking a list haphazard. The important point is that every item (unit, group, class, district) should have the same chance of being included in the selection. The lottery method usually meets this requirement.

Sampling by Conscious Selection.—For sampling by conscious selection groups or classes of items are subjected to a thoughtful selection of representative or typical samples, from which a reasonably reliable indication of the characteristics of the groups may be obtained.

The method may be used for obtaining representative information

(*a*) of various financial aspects of groups of businesses;
(*b*) of production and sales quantities and prices;
(*c*) of personnel.

It is also employed in the selection of data from which various Index Numbers of prices, costs, finance and production are ascertained, as will be described when dealing with this subject. It should be observed, however, that prices used for calculating *general* wholesale and retail price index numbers are largely obtained by random selection.

Practical Examples of Random Sampling.

(a) *Deliveries of coal to a large undertaking.* Figures for every third or fifth barge or wagon can be selected and a sample drawn from the top at each end and from the bottom of each barge or wagon selected.

(b) *Weekly budgets of working class families.* Decide upon all the districts largely occupied by working-class families, and mark out into approximately equally populated sections (not necessarily equal areas); divide the total number of budgets required by the number of sections to decide how many samples from each section; obtain answers to questionnaire from one household in each road in the section up to the number allotted.

(c) *An Article manufactured (or bought) in very large quantities.* Take one from each 100 (or 1000) produced or delivered at different times or on different days.

In each of the above examples the samples will be representative of the characteristics of the whole (or "population").

Danger of Biased Selection.—In making planned or conscious selection there is the danger of bias in the choice of samples. When possible the choice should be made so as to include a number of samples of each class or group which is proportional to the total quantity in the respective groups. Very often, however, this idea cannot be carried out owing to the lack of information as to the numbers in a class or group. In such cases random selection becomes necessary, and to ensure good results a larger number of samples must be taken, making a test by a "trial and error" process, *i.e.* a number of samples are drawn and an average is taken. Several tests are made in this way, and if the variations are wide, a greater number of samples is taken and tested again, the procedure being continued until a reasonably constant or stable result has been arrived at. One writer goes so far as to say that when relying on sampling for statistical purposes it is far better to take 5000 or 6000 cases at random, and examine them to see what their characteristics are, than to take 50,000 which are specially chosen.

QUESTIONS.

1. What do you understand by "Random Sampling"?

2. Upon what principles is the method of statistical sampling based?

3. Explain the following principles: statistical regularity; the inertia of large numbers.

4. What is the purpose of sampling? Compare random and conscious selection of samples.

5. What do you understand by sampling? How would you proceed to choose a sample? (*Incorporated Accountants (Final).*)

6. State and explain the Law of Statistical Regularity. What is its corollary? (*Incorporated Accountants (Final).*)

7. What methods of selection would you use to choose a random sample of: (*a*) Working-class families for purposes of a Cost of Living Index. (*b*) Deliveries of coal to a large factory, and (*c*) A mass-produced manufactured product? (*Incorporated Accountants (Final).*)

8. What are the advantages and disadvantages of the *Sample* method of statistical inquiry? (*Institute of Transport.*)

CHAPTER V

PRESENTING STATISTICAL INFORMATION

Business Statistics has been considered so far from the point of view of the collection of data. There remains to be considered the modes of presentation, which may be :

(*a*) The presentation by textual, tabular, or graphic methods, and

(*b*) The analysis and interpretation of the statistics compiled.

Presentation of Data. It will be obvious that data may be presented in various ways, and the form of the presentation will depend upon the purpose for which the information is desired.

Frequently a simple tabular form is adopted, particularly when the data are required for further use, as for instance preparatory to the drawing of a graphic chart.

On the other hand, a more detailed and planned columnar table may be necessary when an analysis and interpretation of the data are necessary for the purpose in view. Special arrangement and grouping may be called for, and also the inclusion of averages or percentages. In some cases only selected groups or classes of data collected may be included, as greater effectiveness is often secured by the omission of facts unnecessary for the purpose of the report. Care has to be exercised to see that effectiveness and reliability are not jeopardised for convenience of presentation.

A Title should be given to every statistical report, table or graph.

Whenever possible the title should be brief, but clearness should not be sacrificed to brevity.

When a rather lengthy title is unavoidable it is better to split it up into phrases, preferably on separate lines, as for example :—

Eagle Coach Works, Ltd.
Labour Statistics.
Earnings, hours and Efficiencies
by Shops and Grades
Quarter ending, 31st December, 1941.

The titles used on various forms and reports in this book will afford other examples for study.

Choice of Form.—In business the method most easily understood by the person for whose information the statistics have been prepared should be used. Some executives understand and prefer graphs, others can appreciate the information better in tabular form.

Regard must be had to the manner in which the form will be consulted, for instance :—

(*a*) Some managers prefer a large-scale table or graph to hang on the wall.

(*b*) Some users retain all statistical reports in loose-leaf binders, and the size of paper used must be capable of being filed in a uniform size.

(*c*) The use of more than one colour to emphasise different features may be useful.

(*d*) If blue-print, photographic or printed copies have to be reproduced, colours will generally have to be avoided, and lines, letters and figures need to be clean and distinct to ensure good reproduction.

It is not uncommon for both tabular and graphic presentation to be required.

For some purposes a few salient figures may be presented in a textual report with more effect than in other modes of presentation.

Explanatory Notes and Footnotes should be avoided if the facts can be adequately presented without them.

For business use the notes necessary will usually consist of (*a*) Indications as to the limitations of the data, *e.g.* the extent of any approximations, such as figures to the nearest thousand, the nearest hour of labour, decimals correct to the second place, etc. (*b*) A statement as to the type of average shown. (*c*) A note necessary to ensure proper interpretation of any data which would otherwise be capable of being mis-read or from which incorrect deductions might otherwise be made.

Explanatory notes refer to the statement as a whole, but sometimes footnotes are used to clarify specific items in the statement.

When the statistics are for some general use, and particularly when they are likely to be used by others unconnected with the business, explanatory notes are often added (*a*) to give reference to the source of the data used; (*b*) to explain some feature connected with the basis of the compilation; (*c*) to indicate how computations have been made; (*d*) to state what type of average (arithmetic, geometric, median, etc.) is used; (*e*) to draw attention to any limitation of the sources or of the original data necessary to prevent misapprehensions, or to indicate any particular features necessary to a clear interpretation of the facts presented.

Textual Presentation refers to the presentation of statistical facts in paragraph form instead of in the form of tables or graphs. It is a form of introducing statistics to illustrate a written report or survey. The data are thus part of a descriptive discussion, or are simply presented in paragraph form, without comment.

Two methods of arrangement are used:

(*a*) Data are introduced into a textual discussion or description, not in any systematic order, but at convenient points. The two following excerpts are examples:

> "(i) The number of contracts placed last year was 165, making the total to date 640. The value of last year's contracts was £688,548, bringing the aggregate up to £3,603,564, of which £3,250,866 had actually been paid. The total expenditure since the Commission began to operate is £3,279,591. This includes administrative expenses, which in 1935 amounted to £6400, or 0·865 per cent. of the year's outlays."
>
> (ii) Comments on an analysis of 1000 household budgets:
>
> "Altogether there were 612 families, including 1510 adults and 1930 children, with less than 4*s.* a head a week for food, and of these families 156 were receiving unemployment benefit and 230 unemployment assistance, 162 were living on wages, and 64 on public assistance, etc. These 612 families included 78 per cent. of the children."

(*b*) Data are presented in a continuous running form, with or without groupings and systematic arrangement:

> "The sales by the Company's factories have been: 1925, Leeds £10,400, London £25,800, Glasgow £12,000, total £48,200; 1930, Leeds £15,750, London £42,500, Glasgow £17,250, total £75,500; 1935, Leeds £18,900, London £51,300, Glasgow £21,150, total £91,350, showing continuous progress in every instance."

Advantages and Disadvantages.

(a) *Advantages.* Explanatory or descriptive business reports or memoranda incorporating textual data, such as a few important averages or totals, are often more emphatic and convey facts with greater directness and clarity than would be obtained by submitting the same information in tabular or graphic form. Textual comments cannot be overlooked when reading.

The method is often the most simple means of presenting for review certain regularly compiled continuous statistics in which periodic variations are few and not extreme. The regular evenness of figures may result, when looked at in a hurry, in a few variations being overlooked. This type of statistical report often arises in the case of weekly and monthly information connected with commercial and industrial undertakings.

Even when the construction of tabular reports is essential and more convenient for purposes of compilation, the textual presentation of particular variations of interest is often more effective and convenient for the person who has to consider the facts under review.

The method involves less time for both preparer and reviewer, and is therefore economical when it can be used. Instead of the user having to go through a full table of facts he is put directly in touch with the pertinent information. Further, facts are more clearly understood by those who find difficulty in interpreting facts presented in tabular and graphic form.

(b) *Disadvantages.* Except when the data are few, the textual method fails to give an effective and speedy view of the information presented.

This is particularly so when presenting comparative figures relating to detailed classifications. A series or several series of data which are to be compared with other data would be cumbersome and difficult to comprehend if set out in paragraph form instead of in a suitably planned table or chart.

Again, there is the possibility of unintentional, or even deliberate, selection of facts favourable to a certain point of view. The presentation of full statistics in a non-textual manner makes such biassed selection more difficult.

QUESTIONS

1. Name three broad classes of methods of presenting statistical data. What should determine the choice of method ?

2. What are the advantages and disadvantages of textual statistics ?

3. In what form should business statistics be presented ? Mention three forms of presenting statistical information.

CHAPTER VI

TABULAR PRESENTATION

THE arrangement of data tabulations requires considerable thought to ensure showing the relationship between the data of one or more series, as well as the significance of all the figures given in the classification adopted.

Professor Bowley in his *Manual of Statistics* refers to tabulation as "the intermediate process between the accumulation of data, in whatever form they are obtained, and the final reasoned account of the results shown by the statistics."

It will be obvious that the arrangement or layout of the tables is dependent upon the information to be presented, and upon the purpose for which they are to be prepared, but, notwithstanding these points, there are a number of features about tabulations of general application which the student should be acquainted with, and which will now be considered.

The Form of Tabulations.—A few observations on the form of the tables ordinarily used may be considered with advantage before proceeding to describe the technique of classifying and sorting data preparatory to their actual entry in the tables themselves.

In the first place, a distinction must be made between what may be termed (*a*) Informative or Classifying Tabulations, and (*b*) Interpretative or Derivative Tabulations.

Informative or Classifying Tables are original tables which contain systematically arranged data compiled for record and further use, without any intention of presenting comparisons, relationships or significance of the figures. In other words, they merely provide a convenient means of compiling and preserving data in a form for easy reference, very frequently in chronological order.

A factory pay-roll is a tabulation of this type (see Fig. 1).

WAGES-SHEET OR PAY-ROLL

(N.B.—A separate sheet to each shop is desirable.)

Process :
Department or Shop : ....

Week ending : ....19...

Sheet No. : ......
Week No. : ......

Department Symbol No.	Check No.	Name	Grade or Trade.	Hours.*	Rate.	ORDINARY WAGES.		Extra for Bonus.	Gross Wages.	Deductions, Fines.	NAT. INS. WORKER.		Net Wages Paid.	NAT. INS. EMPLOYER.	
						Time.	Piece-work.				H. & P.	Un.		H. & P.	Un.
										(Separate columns for each item of deduction are provided.)					

* Sometimes a column for each day of the week and total are included.

FIG. 1.—EXAMPLE OF AN INFORMATIVE OR CLASSIFYING TABULATION.

Fig. 2 is an example of such a table, used in connection with Sales Orders received, and contains much information from which useful statistics may be compiled.

Sales Orders

February 193...

Date.	Order Number.	Sales Territory.	Class of Item.	Size.	Quantity, Gross.	Price.		Value.		
						s.	*d.*	£	*s.*	*d.*
July 1	901	N. East	C. 12	24	10	10	0	5	0	0
	902	London	D. 36	01	24	20	0	24	0	0
	903	South	C. 12	12	10	7	6	3	15	0
,, 2	904	London	C. 12	24	5	10	0	2	10	0
	905	N. West	A. 23	12	6	40	0	12	0	0
etc.										

FIG. 2.—AN INFORMATIVE OR CLASSIFYING TABLE.

Interpretative or Derivative Tables are distinct in that they are analytical and are prepared to present significant aspects of the data comprised.

This type of tabulation may be : (*a*) Simple ; (*b*) Complex.

(a) *A Simple Table* presents the number or measurement of a single set of items having the characteristics stated at the head of a column or row which forms the basis of the table. Using the terminology of statistics, we may tabulate a dependent variable (*e.g.* output of production) against an independent variable (*e.g.* time) as shown in Fig. 3.

The Irrigation Company

Annual Production

1931–1937

Year.	Production Output.
	£
1931	2,750
1932	4,600
1933	6,450
1934	7,200
1935	8,450
1936	9,080
1937	10,500

FIG. 3.—A SIMPLE INTERPRETATIVE OR DERIVATIVE TABULATION.

Another example is given in Fig. 4 in which the dependent variable (number of families) is tabulated against the independent variable (amount a head for food).

Food Expenditure of Families in Receipt of Wages

Based on analysis of budgets of 358 households.

Amount per head for Food.	Number of Families.
2*s*. 6*d*. to 3*s*.	58
3*s*. 6*d*. to 4*s*.	96
4*s*. 6*d*. to 5*s*.	82
5*s*. 6*d*. to 6*s*.	52
6*s*. 6*d*. to 7*s*.	30
7*s*. 6*d*. to 8*s*.	40
Total 2*s*. 6*d*. to 8*s*.	358

FIG. 4. A SIMPLE INTERPRETATIVE OR DERIVATIVE TABULATION.

(b) *A Complex Table* presents the number or measurement of more than one group of items set out in additional columns or rows, and often the table is divided into sections.

Such tables generally show the relationship of one set of data to another or others, and are often so arranged that comparisons may be made easily between related facts.

An example of a complex table is given in Fig. 5. It is rather a full table, and the information might be made more effective by dividing it into two or even more separate tabulations. A less involved tabulation is that given in Fig. 6.

In the case of business statistics it is more usual to find complex tabulations, because of the incorporation of full information to facilitate a proper consideration of all related facts. Comparative figures, whether absolute, or percentages, or averages of various kinds, are frequently required and are therefore incorporated in many forms of tabulated statistics. See for example Fig. 7, taken from the author's *Cost Accounting and Costing Methods*, and Fig. 97 on p. 176.

QUESTIONS

1. What do you understand by the following: (*a*) Classifying Table; (*b*) Derivative Table; (*c*) Simple Table; (*d*) Complex Table?

2. What types of tabulation are used for business statistics and how are they compiled?

3. Draw up a simple tabulation to show the production output of a factory department based on monthly reports.

FACTORY LABOUR AND WAGES STATISTICS

Week ending 5th May, 19.....

Department.		No. of Employees.		Hours Worked						Piecework.				Allocation of Gross Wages.											
No.	Name.	Working.		Total	Per Person.		Gross Wages with Piecework.			Amount Paid			% Efficiency.	Direct.			Indirect.			Non-Manufacturing.			% Direct to Total.	% Indirect to Total.	% Non-Mfg. to Total.
		Male.	Female.		Male.	Female.	£	s.	d.	£	s.	d.		£	s.	d.	£	s.	d.	£	s.	d.			
410	Foundry	21	9	1,381·3	46	45	51	17	11½	2	10	2	98	36	6	10	10	11	10½	4	19	3	70	20	10
413	Plating	2	26	1,231·8	46	44	33	6	3			1	106	30	14	1		2	9	2	9	5	92	1	7
414	Enamelling	1	7	387·2	47	47	14	9	2	2	1	2	95	10	10	0	1	3	7	2	15	7	73	8	19
415	Polishing	16	10	1,175·2	44	47	58	15	9½	15	17	1	99	47	13	8	7	12	11½	3	9	4	81	13	6
416	Machine (A)	65	27	4,434·2	51	47	202	1	3	13	16	3	96	84	1	5	78	15	4	39	4	6	42	39	19
421	Winding	2	8	457·4	49	47	17	18	5	2	5	3	97	11	18	5		8	0	5	11	10	67	2	31
433	Fitters	49	3	2,432·5	46	45	136	19	7	30	9	4	99	99	4	1	24	19	10	12	15	8	73	18	9
434	Machine (M)	7	55	2,838·6	48	45	81	19	5½	5	3	6	80	56	9	11	13	3	5½	12	6	1	69	16	15
	Total Manfg. Depts.	183	145	14,338·2	48	46	£596	18	8½	£70	4	10	—	£376	5	3	£136	17	9½	£83	11	5	63	23	14
401	Progress	6	4	467·5	48	47	21	17	9											21	17	9			100
408	Inspection	12	12	1,179·7	53	46	51	3	3½											51	3	3½			100
409	Maintenance	14	2	869·5	53	49	44	2	10											44	2	10			100
412	Despatch	9	4	696·7	57	48	31	3	6											31	3	6			100
419	General Office	12	14	1,132·5	61	46	44	11	10											44	11	10			100
420	Stores	11	8	474	46	48	33	0	3½											33	0	3½			100
424	Experimental	12		783·9	64		48	5	1½	3	2	5	102	6	15	8	35	6	2½	5	4	8	14	75	11
	Total Non-Manfg. Depts.	76	43	6,201·6	54	44	£274	4	7½	£3	2	5		£6	15	8	£36	6	2½	£231	4	2	3	13	84
	Gross Totals	259	188	20,539·8	51	46	£871	3	4	£73	7	3	98	£383	2	11	£173	4	7	£314	15	10	44	20	36

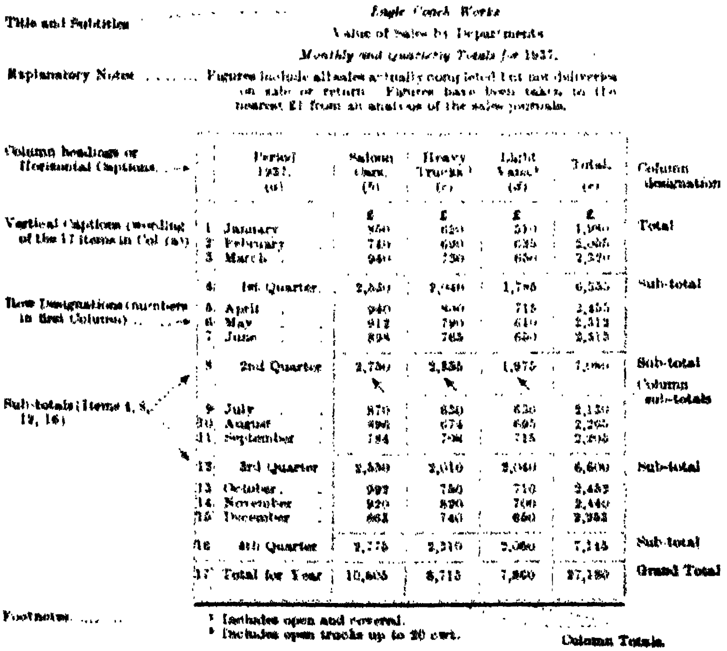
FIG. 5.—AN EXAMPLE OF A COMPLEX TABULATION.

CHAPTER VII

CONSTRUCTION OF TABLES

In this chapter consideration will be given to a number of features and commonly adopted rules which are of general application.

Columnar Layout.—The number of columns and the arrangement or layout of the form must be carefully considered to ensure that the table is not too complicated and confusing.



Title and Subtitles
Explanatory Notes
Column headings or Horizontal Captions
Vertical Captions (wording of the 17 items in Col. (a))
Row Designations (numbers in first Column)
Sub-totals (Items 4, 8, 12, 16)
Footnotes
Column designation
Total
Sub-total
Sub-total
Column sub-totals
Sub-total
Sub-total
Grand Total
Column Totals.

Eagle Coach Works

Value of Sales by Departments

Monthly and Quarterly Totals for 1937.

Figures include all sales actually completed but not deliveries on sale or return. Figures have been taken to the nearest £1 from an analysis of the sales journals.

	Period 1937. (a)	Saloon Cars. (b)	Heavy Trucks[1] (c)	Light Vans[2] (d)	Total. (e)
		£	£	£	£
1	January	850	625	510	1,985
2	February	740	690	635	2,065
3	March	940	730	650	2,320
4	1st Quarter	2,530	2,045	1,795	6,555
5	April	940	800	715	2,455
6	May	912	790	610	2,312
7	June	898	765	650	2,313
8	2nd Quarter	2,750	2,355	1,975	7,080
9	July	870	850	630	2,350
10	August	996	674	605	2,275
11	September	784	704	715	2,203
12	3rd Quarter	2,650	2,010	2,000	6,600
13	October	993	780	710	2,483
14	November	920	820	700	2,440
15	December	862	740	650	2,252
16	4th Quarter	2,775	2,310	2,060	7,215
17	Total for Year	10,605	8,715	7,800	27,180

[1] Includes open and covered.

[2] Includes open trucks up to 20 cwt.

FIG. 6.—COMPLEX TABULATION, ANNOTATED TO EXPLAIN TERMINOLOGY.

Too many columns and subdivisions may make interpretation difficult, and obscure facts which require emphasis.

It may be advisable to prepare subsidiary tables, and a more summarised final tabulation which will present the essential data more clearly and effectively.

The Position of Various Sets of Items in the table requires particular attention.

Ordinarily it is more convenient to arrange the more numerous words and figures vertically rather than horizontally. (See Fig. 6, vertical captions.) As far as possible vertical captions should be condensed to occupy one line, widening the column to permit of this if convenient.

The Size of the Sheet or Space available demands careful drafting of the ruling and spacing of the columns. Usually the procedure is to prepare a rough draft, in order that a neat lay-out, with columns of convenient and balanced width may be decided upon.

Space for the title, footnotes and other references has to be allowed for. Footnotes are very useful for amplifying the horizontal captions at the head of each column, particularly when the columns must be rather narrow.

The Columnar Headings (or horizontal captions) should be as brief as possible, but should be unmistakable. If necessary, words may be divided and the caption run into more than one line. If a brief columnar heading is not sufficiently definitive, it may be amplified by a footnote, although it is advisable to avoid these footnotes whenever possible.

Frequently the column headings refer to some measure or quantity of units, and care should be taken that the unit is unmistakable, *e.g.* do not state just " Length," but " Length in feet " or other unit.

Large numbers can often be curtailed, *e.g.* " In thousands of £'s," or " 000's omitted," as shown in Fig. 9. The fact that a number of digits has been so dropped should be stated in the caption at the head of the column concerned. Much space can be saved and more clarity given to the table by thus tabulating to the nearest million, thousand or hundred, as the case may be.

A heading may be common to more than one column, and when this is the case the space for the heading may be divided horizontally, and the caption run across the columns, as shown in the last six columns of Fig. 5 and in Fig. 7.

STEEL ROLLING-MILL

Summary of Production and Costs

Week ending: ....19...

Department.	Production.		Total Expense. *W.E.:*			(Columns Repeat.)
	Tons.	Hours Worked.	£	Average per		
				Hr.	Ton	
Hot Mill 1						
,, 2						
Total Hot Mills			£			
Cold Mill 1						
,, 3						
,, 4						
Total Cold Mills			£			
Pickling Shop 1						
,, ,, 2						
Total Pickling			£			
Annealing 1						
,, 2						
,, 6						
Total Annealing			£			
Paring and splitting						
Cutting to length						
Shearing						
Hardening and Tempering						
Polishing						
Grinding						
Warehouse						
Delivery Charges						
Total Miscellaneous Processes			£			
Steel Service						
Scrap ,,						
Total Steel Service						
Grand Total			£			

FIG. 7.—EXAMPLE OF A COMPLEX TABULATION SHOWING COMPARATIVE STATISTICS.

Row or Column Designations.—When the table comprises many columns or rows it is useful to number them, as in Fig. 6. This is particularly useful when the table is to be referred to in a textual description or report. It also adds clarity, if, in a long table, there is a break or space after every fifth or tenth line, unless a convenient sub-total, or subheading, can be inserted.

Proximity of Related Figures.—Comparative figures should be placed as close to one another as possible to make comparison easier. Similarly, derivative figures like percentages or averages should be placed as close as possible to the original absolute figures. (See Figs. 5 and 7.)

Effective Rulings.—Rulings may be made very effective by varying them. Major divisions can be separated by heavy or double lines and subdivisions by lighter, finer lines. Subdivision of the space for captions at the head of columns are given a better appearance by the use of fine lines.

Double lines look well under grand totals, and, when used, the upper one should be fine, the lower one heavy. The use of such rulings is illustrated in Figs. 5 and 7.

Printed and typewritten tables can often be set up without the use of ruling, by proper spacing. For instance, major vertical divisions can be separated by a wide space, and related columns can be set closer together, as shown in Fig. 8.

Comparative Prices

		Prices			Proportionate Numbers (percentage ratios)		
		Wheat	Sugar	Tea	Wheat	Sugar	Tea
Average	1870–79	11·5	31·9	16·3	100	100	100
Year	1900	6·8	12·8	8·5	59	40	52
,,	1920	26·9	64·0	15·0	234	201	92
,,	1927	12·3	18·5	18·5	107	57	113

Fig. 8.—Effective Tabulation Without Columnar Rulings.

Position of Totals.—Totals of rows are usually placed in the extreme right column, although they are often placed in the first column after the vertical captions on the left. Totals of columns are usually at the foot, and this is the most convenient arrangement when the totals are used primarily as checks. In many instances published statistical tables present totals at the top of the table, and for many purposes this is very convenient.

Order or Arrangement of Items.—Entries are usually inserted to be read from left to right, and from top to bottom. It is for this reason that totals of columns and the grand total usually appear at the foot of the table.

The principal arrangements of items are alphabetical, chronological, geographical, size or importance, order of interest or emphasis, and sometimes according to causal relationship to facilitate comparison.

Data in chronological order which are to be compared should usually be tabulated vertically, with comparative or related figures shown in adjacent columns. An inspection of the two tables set out in Figs. 9 and 10, is convincing that it is easier to follow the figures as shown in Fig. 9 than when arranged horizontally as in Fig. 10.

Comparison of Production and Sales

1930-1936

Year.	Production (000's omitted).	Sales (000's omitted).
1930	3275	2800
1931	3720	3250
1932	4100	3800
1933	4905	4160
1934	5300	4930
1935	6750	6680
1936	7120	7200

FIG. 9.—VERTICAL ARRANGEMENT OF ITEMS.

(000's omitted).	1930.	1931.	1932.	1933.	1934.	1935.	1936.
Production .	3275	3720	4100	4905	5300	6750	7120
Sales . .	2800	3250	3800	4160	4930	6680	7200

FIG. 10.—HORIZONTAL ARRANGEMENT OF ITEMS.

Methods of Giving Emphasis.—Should it be desired to emphasise some of the items or groups this may be done in one or more of the following ways :—

(*a*) By distinctive type, *e.g.* italic or heavy type if printed; or red ink in written tables. (See pages 103 and 111.)

(*b*) By special marking, such as underlining or a " box " or circle round an entry.

(*c*) By use of distinctive lines, heavy and light, or single and double, between sets of figures. (See pages 32, 193 and 195.)

(*d*) By the order of entry in the table, *e.g.* date order, order of magnitude.

(*e*) When the table is long, by leaving a space between every group of say five lines. (See page 196.)

Comparative figures, ratios, percentages and averages, estimates as contrasted with actual figures, and minus quantities, can all be shown in printed statistics by means of italic type. In hand-written tables coloured ink may be used instead.

QUESTIONS

1. What matters would you consider when about to construct a statistical Table?

2. In what way may emphasis be given to items in a tabulation?

3. Put the following statement into tabular form, paying special attention to headings, showing the changes between the two dates, and giving columns in which percentages of the appropriate totals can be entered. Calculate and fill in the percentage in each year for the Plate and Jewellery Trades.

"In 1930 the value of the net output of the Copper and Brass Trades in Great Britain was £5·8 Mn.; of the Finished Brass Trade, £5·7 Mn.; of the Lead, Tin, Aluminium, etc., Trades, £6·2 Mn.; of the Gold and Silver Refining Trades, £1·1 Mn.; of the Plate and Jewellery Trades, £4·6 Mn.; and of the Watch and Clock Trades, £0·5 Mn. For 1935 the values were, respectively, £6·9 Mn., £6·6 Mn., £9·8 Mn., £1·0 Mn., £4·4 Mn., and £0·6 Mn." (*N.A.L.G.O.*)

4. What decisions should be taken before preparing a statistical table? (*Incorporated Accountants (Final).*)

5. State in the form of a list the considerations which would be in your mind in framing a table of statistics. (*Incorporated Accountants (Final).*)

6. State the considerations you would observe in constructing a statistical table. (*Incorporated Accountants (Final).*)

7. Construct a simple table (using your own illustrative figures) showing:—Total net tonnage of British and Foreign Vessels, distinguishing sailing and steam (including Motors) entered and cleared, in the foreign trade at ports in the United Kingdom with cargoes and in ballast. (*Incorporated Accountants (Final).*)

8. Draw up in blank a table in which could be shown for a certain area the distribution of private families according to numbers of persons in family and the size of dwelling occupied in terms of numbers of rooms, totals and sub-totals, and the percentages which the sub-totals bear to their respective totals. Fill in one column and one row of illustrative figures, so as to show how your table should be read. Give full headings, etc. (*N.A.L.G.O.*)

CHAPTER VIII

PREPARATION OF FIGURES FOR TABULATION

THIS necessitates making a selection of the classes or groups, and of the headings and subheadings to be used.

Generally working sheets will be required, and care should be exercised to avoid errors :—

Figures should be clearly written, allowing plenty of space.

The headings selected should be shown on the sheets to avoid mixing data by mistake.

Arrange some sort of check on the calculations and totalling.

When possible cross-cast checks should be provided.

When sorting and counting are necessary tally sheets are often useful. When counting numerous sorted data it is usual to mark strokes on the working paper in fives arranged thus :—

Number of Wage-Earners at Various Hourly Rates

		Total.
8*d*.-9*d*.	𝍸 𝍸 𝍸 |||	18
10*d*.-12*d*.	𝍸 𝍸 𝍸 𝍸 𝍸 ||||	29
13*d*.-15*d*.	𝍸 𝍸 𝍸 ||	17

The groups of five are then easy to count.

Mechanical Aids.—Various makes of machines are available for adding and listing.

Sorting data into classes or groups may be a very lengthy process, especially when the same facts have to be sorted and re-sorted into several different classifications. To facilitate this, much time can be saved by transferring data to special sorting cards. The time spent in preparing the sorting cards will save much more time in the end.

Needle-Sorting by Hand, of which the Paramount System * is a well-established method, provides a practical means of sorting large numbers of items under as many headings as may be desired in an economical manner.

---

* Patented by the Copeland-Chatterton Co., Ltd., Exchange House, Old Change, London, E.C.4.

Cards of any convenient size or shape with a series of holes punched along the extreme edge or edges are used. Each hole stands for a number or value, or other detail in relation to the headings grouped on the card. Codes are used to reduce the number of holes. (See Fig. 16*a*, page 43.)

The value of the system is that original facts are written direct on to the cards thus saving time and avoiding the possibility of error when data have to be transferred from original records on to the cards. The holes along the edge referring to the numbers representing the written facts are snipped out (thus making a slot) with a pair of nippers.

When a stack of cards has been so slotted a long needle is passed through a particular hole representing a certain number or fact, then, with the cards standing on their edges, when the needle is raised as if to lift all the cards in the batch, those cards on which the desired information appears drop out, and the rest of the cards remain on the needle. (See Fig. 16*b*). The procedure is repeated as often as required to obtain from the batch all the cards for each item of data. The cards left behind, after lifting away the cards in which the hole is intact, are the only ones containing the particular number or fact it is desired to sort out, and they may therefore be counted. If desired, they can be listed and totalled on a listing and adding machine, of which there are several on the market, such as the Burroughs. For general business purposes the Paramount System is the quickest method, as cards can be sorted at speeds up to 60,000 per hour. It provides a visible check against mis-sorting, as will be seen in Fig. 16c. The slotting of the holes creates channels in the pack of cards, and the bigger the classification the longer the channel. An incorrectly placed card breaks the channel. It is a self-evident and immediate check on error.

Mechanical Sorting.

Punched Cards.—It is necessary to have the original data on special cards of uniform shape and size. Whether written directly on the special cards, or taken from other records, the data are recorded on the special cards by punched holes made by means of a special *key punch* operated by hand or electrically. (See Fig. 11.)

For this purpose the information is suitably coded numerically. For instance, a code for stores data in a business could be a six-figure one, divided into two sections, the first three figures representing the group or class, and the second three figures the item in the group. With such a code the stock can be divided into 999 groups, each with a capacity of 999 items.

The Hollerith cards, for use with the Hollerith Sorting and Tabulating Machines, are of uniform size (7⅜" × 3¼"), and are printed with 45 or 80 vertical columns of numerals, 0 to 9 (Fig. 11). The Powers-Samas cards, for use with the standard Powers-Samas Sorting and Tabulating Machines, are similar, but have 45 to 90 columns. The Powers-Four cards are 2" × 4½", with only 26 columns for use in their smaller model machine, which is useful for businesses not requiring the capacity of the standard sorter.

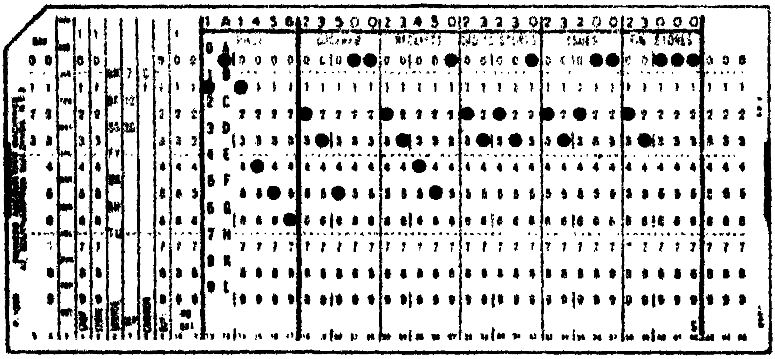


FIG. 11.—EXAMPLE OF PUNCHED CARD FOR MECHANICAL SORTING.

MASTER PRODUCTION CONTROL CARD

Cols. 1–3. Date.
4. Shop number.
5. Store number.
6. Source (bought out rough, bought out finished, stamp, foundry, bar, sheet and turn).
7. Common (to designate if piece is common to 7 H.P., 12 H.P. and 20 H.P. chassis).
10–11. Number required on each chassis (*e.g.* 1 if propeller shafts, 4 if piston, etc.).
12–17. Piece.
18–22. Quantity (in terms of chassis number) ordered.
23–27. Receipts (in terms of chassis number) on receiving dock.
28–32. Deliveries to Stores (in chassis number).
33–37. Issues to production (in chassis number).
38–42. Return from production to finished store (in chassis number).

Cards of the same design and wording, but of different colour, are used for detail additions and deductions.

In order to check the accuracy of the punchings on the cards a verifying key punch is used. Should an error have been made in punching, the wrongly punched card remains stationary in the verifying machine, thus compelling the operator's attention to the mistake.

FIG. 12.—KEY PUNCH FOR PERFORATING CARDS (HOLLERITH).

The punching can be done at a rate of 200 to 400 cards per hour. Sometimes much of the information recorded is common to a large number of cards, and much time can be saved by using an electrical duplicating punch. A master card is prepared for the items to be duplicated and with this inserted this special punch automatically punches and ejects the cards, which are fed in from a magazine. When this duplicated punching has been completed the rest of the punching is done by the hand punch in the ordinary way.

A Hollerith key punch is shown in Fig. 12.

The card illustrated in Fig. 11, taken from *Cost Accounting and Costing Methods*, shows a card divided into sections by thin vertical lines. The space between each pair is termed "a field." The information punched on the card (holes punched are represented by black spots) reads as follows :—

Field 12–17.	Reference number of article . .	1A1456
,, 18–22.	Quantity ordered	23,500
,, 23–27.	,, received at works . .	23,450
,, 28–32.	,, delivered to store . .	23,230
,, 33–37.	,, issued to production .	23,200
,, 38–42.	,, returned from production department to finished store .	23,000

The forty-five column card may be divided into convenient fields suitable to the purpose in view, thus, for Sales Data the arrangement might be as follows :—

Date of invoice	3	columns.
Serial Number	5	,,
Traveller's Area	2	,,
Town	3	,,
Type of business	3	,,
Customer	4	,,
Article	3	,,
Quantity (Weight, No.)	6	,,
Price per Unit	5	,,
Amount charged	8	,,
Department or Ledger	3	,,
	45	

A division of the card for production purposes is indicated in Fig. 11.

Sorting is done electrically after the punching has been completed and verified. Once the cards are punched the data can be sorted into any desired order or classification, *e.g.* departments, travellers, article numbers and quantities, etc.

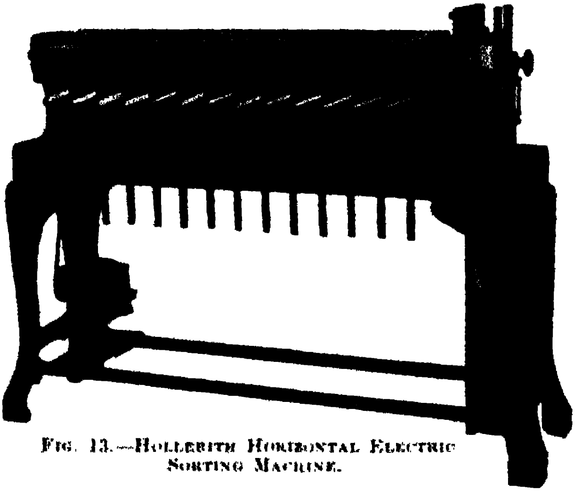
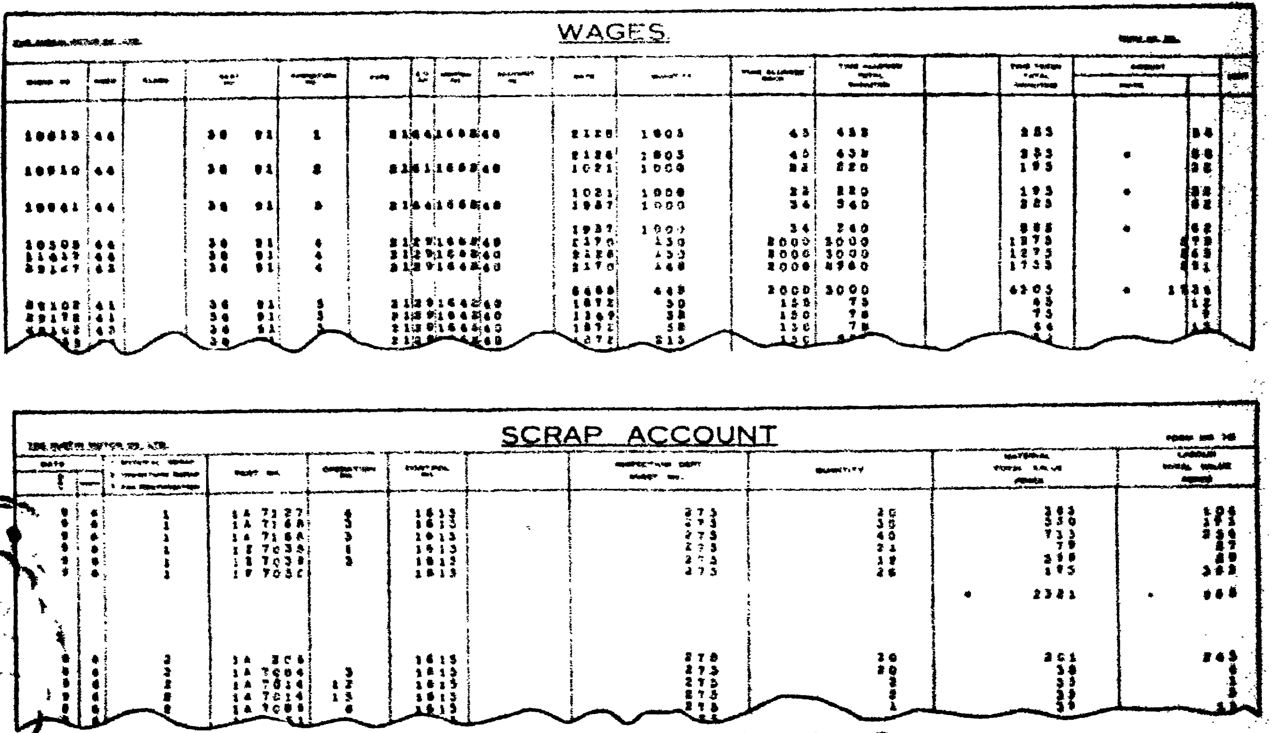


FIG. 13.—HOLLERITH HORIZONTAL ELECTRIC SORTING MACHINE.

Both the Hollerith and the Powers-Samas sorting machines sort the cards at the rate of about 24,000 per hour. The cards are placed in a stack, a pointer is set to the column to be sorted



WAGES.

SCRAP ACCOUNT

FIG. 14.—SPECIMENS OF TABULATIONS PRINTED BY A POWERS-SAMAS TABULATOR.

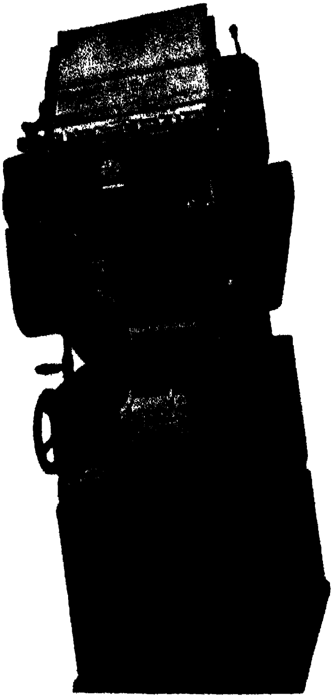


FIG. 15.—POWERS-SAMAS ALPHABETICAL PRINTING TABULATOR.

and as the cards pass into the machine electric contacts are made through the punched holes, and automatically the cards are moved along and dropped into the various sorting pockets or slots. (See Fig. 13.)

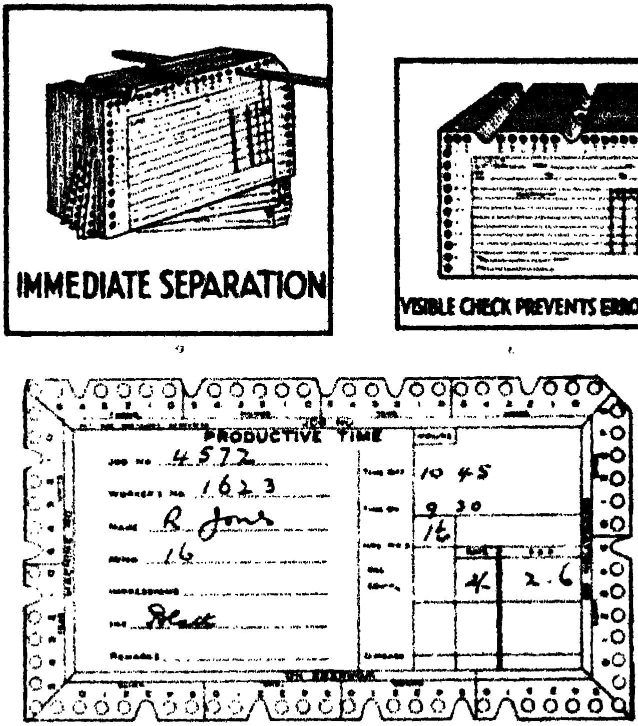


FIG. 16.—PARAMOUNT NEEDLE SORTING SYSTEM: (a) TYPE OF CARD; (b) METHOD IN OPERATION; (c) A CLEAR CHANNEL, AND ONE SHOWING ONE MIS-SORTED CARD.

Tabulating.—The *sorted* cards are then placed in the tabulating machine, which (*a*) lists and adds all the information desired on every card, item by item, (*b*) accumulates sub-totals and grand-totals of the items and (*c*) counts the number of cards as they pass through the machine. The tabulated information is printed on to sheets, an example of which, from *Cost Accounting and Costing Methods*, is shown in Fig. 14. Illustrations of the Powers-Samas and Hollerith machines are given in Figs. 15 and 17.

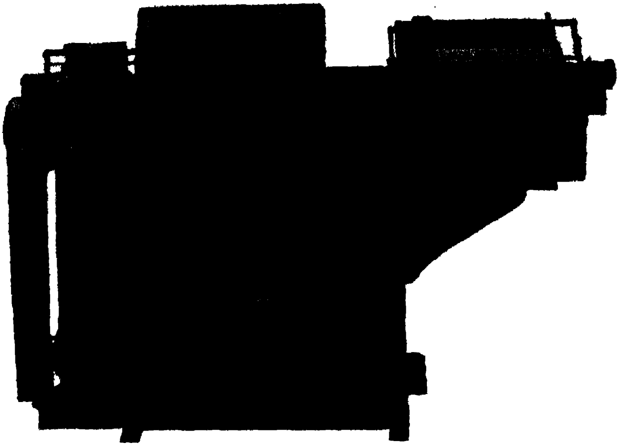


FIG. 17.—HOLLERITH PRINTING 38-COLUMN ELECTRIC TABULATOR.

QUESTIONS

1. What mechanical aids may be used for sorting data?

2. Explain "needle sorting."

3. A company manufacturing some 700 different products desires to take out significant statistics of its sales in the United Kingdom using a numerical code for each characteristic selected, and performing the operations of classification by use of Hollerith Machines. The Hollerith card contains 45 columns of figures, each running downwards vertically from 1 to 9. State the characteristics you would select, and show how you would use the columns on the card to the best advantage. (*Incorporated Accountants* (*Final*).)

4. State the checks you would impose to verify accuracy in tabulation. (*Incorporated Accountants* (*Final*).)

CHAPTER IX

GRAPHIC METHODS

I. Diagrams and Pictograms

Diagrams and Graphs.—Instead of tabular presentation of statistical data a number of other devices are employed. Some business men find it difficult to grasp statistical relationships from columns and tables of figures, or prefer to have the meaning of groups of figures presented at a glance, and for this purpose the following methods are available :—

(*a*) Pictograms, *i.e.* pictures and diagrams to show relative or proportional sizes, etc.
(*b*) Cartograms, or maps variously marked to show geographical distribution or density.
(*c*) Simple bar-charts.
(*d*) Multiple bar-charts.
(*e*) Component bar-charts.
(*f*) Percentage diagrams and charts, which may be (i) bar-charts, or (ii) pie- or circular charts.
(*g*) Curve charts or graphs, which may be considered in two groups :—
    (i) Arithmetic or natural scale graphs.
    (ii) Ratio graphs, often called semi-logarithmic graphs, drawn on a ratio scale.

Pictograms, consisting of diagrams drawn in different sizes to scale, are sometimes used to demonstrate relative sizes. For instance, the output of two factories, one of which was twice as great as that of the other, might be illustrated by a diagrammatic sketch of two cars, one being half the size of the other. The method is useful to show readily the significance of simple classes of data. It is a method which can be understood by those who might find it difficult to interpret the same data in a table or curve-chart.

Other forms of this type of presentation for comparison purposes are squares, rectangles, cubes, etc. It will be apparent that comparison of magnitude is easier to judge from diagrams of one dimension (bars or lines) than from diagrams which vary in two or three dimensions (areas and volumes).

The method is not satisfactory for serious statistical comparisons owing to the difficulty of accurately measuring with the eye; it is popular, however, with advertisers.

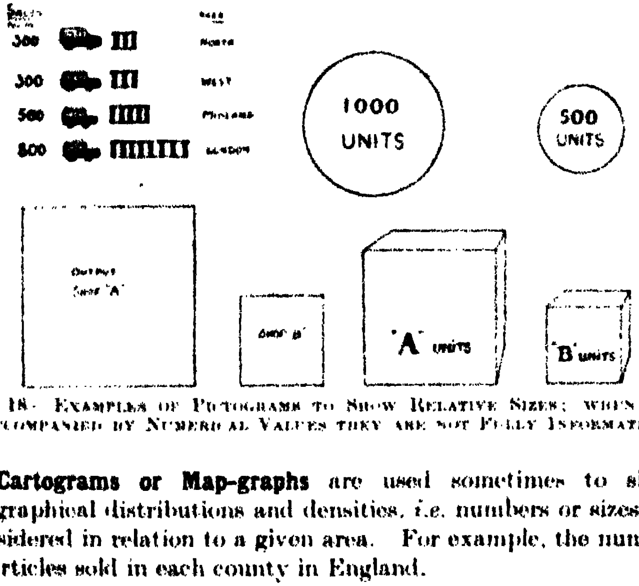


FIG. 18.—EXAMPLES OF PICTOGRAMS TO SHOW RELATIVE SIZES; WHEN NOT ACCOMPANIED BY NUMERICAL VALUES THEY ARE NOT FULLY INFORMATIVE.

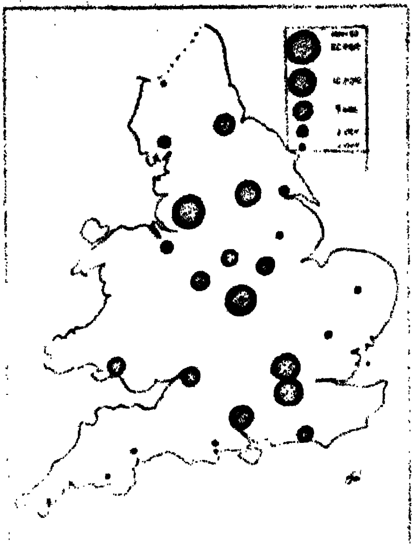
Cartograms or Map-graphs are used sometimes to show geographical distributions and densities, *i.e.* numbers or sizes are considered in relation to a given area. For example, the number of articles sold in each county in England.

A map of the area under review is marked in a way which conveys to the eye a comprehensive impression of distribution. The devices employed are as follows, taking as an illustration the quantity of a manufacturer's machines calculated to be in use in England; salesmen's territories :—

(*a*) Dots proportional in size to the quantity in each area (Fig. 19).

(*b*) Dots of equal size, each representing, say, 1000 machines. In districts where there are several thousands in use the density of the dots will be greater (Fig. 20).

(*c*) Cross-hatching may be used to indicate densities. The greater the density of use of the machines the closer the lines of the cross-hatching (Fig. 21). Other forms of shading (*e.g.* dots) or colouring may be similarly used.



Machines in Use in Salesman's Territories.

FIG. 19.—CARTOGRAM SHOWING DISTRIBUTION.

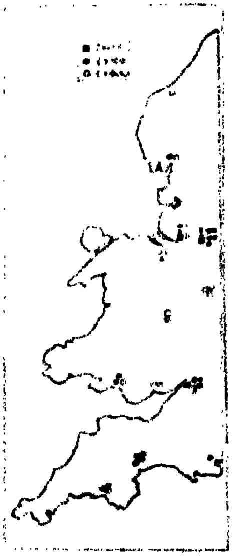


FIG. 20.—CARTOGRAM SHOWING DISTRIBUTION OF SALES ORDERS BY TERRITORIES.

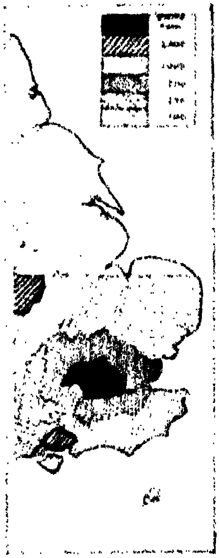


FIG. 21.—CARTOGRAM; VARIOUS METHODS OF SHOWING DENSITY OF USE OF PRODUCTS.

Bar-Charts (Simple).—The use of simple bar-charts is one of the commonest ways of indicating simple comparisons of size. These charts take the form shown in Figs. 22 and 23.

The choice of horizontal or vertical depends upon the purpose. When the items in a group are to be compared, it is optional whether the bars are drawn horizontally or vertically (Fig. 22).

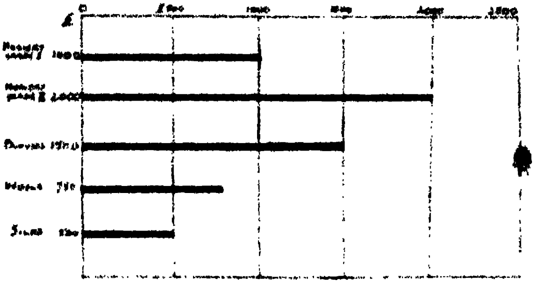


FIG. 22.—HORIZONTAL BAR-CHART COMPARING RELATIVE SIZES OF SALES BY-PRODUCTS.

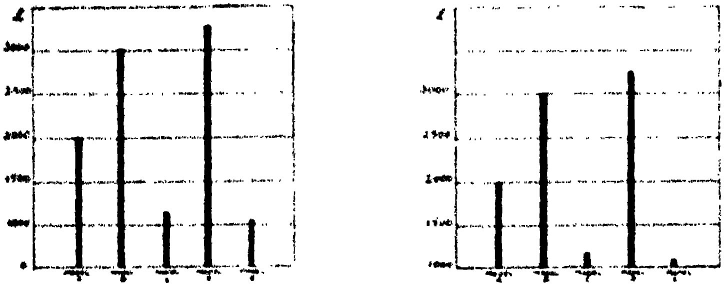


FIG. 23.—BAR-CHART DRAWN CORRECTLY, SHOWING THE BASE LINE.

FIG. 24.—BAR-CHART DRAWN INCORRECTLY, AS IT DOES NOT SHOW A ZERO BASE LINE.

When, however, the data are to be charted with reference to a series of dates, the vertical form is usually used, with the time scale along the base line.

The bars are more informative if drawn to a given scale, and it is very important that when such charts are used for comparisons of size the *scale must begin at zero*, otherwise a false impression is given. The false idea of proportions is clearly shown by Figs. 23 and 24.

Hints on Making the Chart

(i) The numbers on the scale should not be too numerous, and should run in intervals easy to follow, *e.g.* 2's, 5's, 10's, 100's, etc. The scale must, of course, be suitable for the information to be charted.

(ii) When the actual figures represented by the bars are required to be shown, they should be inserted as in Figs. 22 and 23. Avoid placing them along, or at the free ends of the bars themselves.

(iii) Squared paper, commonly known as co-ordinate paper, is usually very convenient, but if plain paper is used, a few fine guide-lines to assist the eye should be inserted as a background.

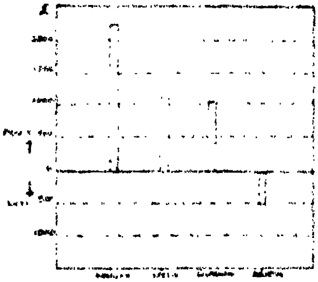


FIG. 25.—BAR CHART; A LOSS IS SHOWN BY THE BAR BELOW THE ZERO LINE.

(iv) When bars are broad and shaded or coloured, the guide-lines should not run through the bars, but just up to them (Fig. 26).

(v) Neatly printed figures and letters should be used; long-hand figures, titles and descriptions seldom give a good appearance. Ornate lettering is inadvisable for business charts. Confidence is given when a chart is neatly drawn, well proportioned and carefully spaced.

(vi) The title is best placed at the top where business men are used to finding headings, and it should be the largest lettering on the chart, so that its purpose is clearly appreciated immediately.

(vii) The representation of losses and negative items should be made by bars continued below the zero line (Fig. 25)

The Use of Simple Bar-Charts.—These charts are very useful to represent graphically many kinds of business data, such as :—

Quantities or Values of different products bought, sold or produced, etc.

Data concerning employees.

Expenses by departments, branches, depots, etc.

Profits by branches or subsidiaries, or by lines of product.

Comparisons between budget and actual figures.

Progress of work in production or of stock movements, and so on.

Production of Three Types of Motor Vehicles

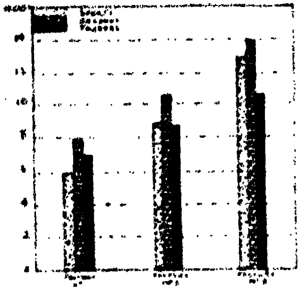


FIG. 26.—MULTIPLE OR COMPOUND BAR CHART FOR SHOWING COMPARISON OF OUTPUTS

Progress Charts provide a practical example of the use of a simple horizontal bar chart. This type of chart is useful for keeping in close touch with production, sales and other data of importance to the business executive. For instance, in relation to production the schedule of work is represented by a horizontal line, with quantities budgeted for at each point of time. As the work proceeds, another bar, drawn thicker or in another colour, is run parallel to the scheduled bar. If quantities produced are less than those scheduled, the line will fall short of the line representing scheduled quantities. When the weekly (or daily) schedule is exceeded, the excess is drawn above, as in the third week in Fig. 27. A progress chart of the following facts is shown in Fig. 27.

	Scheduled.	Completed.
1st week . . .	80 Units	70 Units
2nd ,, . . .	70 ,,	80 ,,
3rd ,, . . .	100 ,,	130 ,,
4th ,, . . .	90 ,,	80 ,,
Total . . .	340 Units	360 Units

The fine lines and the numbers to the left of them represent scheduled work. The numbers on the right are the progressive totals. The thick lines show the actual work done each week. The fine, continuous line represents the scheduled work for four weeks (340 units). The thick line below this is the actual work done in the four weeks (360), *i.e.* twenty units more than scheduled.

This type of chart is one which has been very extensively developed by H. L. Gantt, and it is usually called the Gantt Chart.

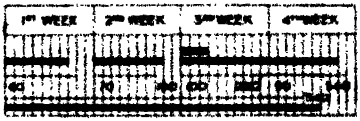


FIG. 27.—EXAMPLE OF A PROGRESS CHART (BAR TYPE).

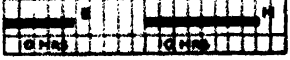
A number of different facts may be represented on one chart, by using different thicknesses of line, or by different coloured inks. The former method is necessary, if blue prints are to be made of the chart for circulation in the business to executives.

The Uses of Progress Charts for Manufacturers.—The following remarks have particular reference to the Gantt type, which is useful in manufacturing concerns for:—

(1) All comparisons of scheduled work and time with actual, and to show progress, and to serve as an incentive to maintain lay-out of production; and for showing,

(2) Running times of machines possible as compared with actual. Any space at the end of the thick line, representing actual, can be explained by code letters and footnotes, thus:—

Daily Running Time of Machines.



B. Machine breakdown. M. Waiting for materials.

FIG. 28.—USE OF CODE LETTERS ON PROGRESS CHART (GANTT CHART).

(3) Labour hours can be similarly treated, giving reasons for short time, etc. A separate chart for each man can be drawn for the use of the foreman; or, for each shop, for the manager to check the foreman's results. Lettering can be

used to indicate absence, slow workers, sickness, accidents, tool or machine breakage, faulty material, waiting for orders, etc.

(4) The load, or number of orders, or quantity of work, assigned to shops, machines or men, can be indicated to ensure even distribution.

(5) For sales by quantities or values by individual salesmen or departments. Budgeted or planned results are shown by a bar, whilst another bar is drawn to represent figures actually realised both consecutively and cumulatively.

(6) A variety of useful data concerning personnel and financial and manufacturing operations can be effectively shown at a glance.

Multiple or Compound Bar-Charts are the same in principle as those already described, but are designed to show two or more sets of data by grouping several bars together to facilitate comparison. The arrangement necessitates each bar in the group being identified by distinctive shading or colouring as shown in Fig. 26.

Component Bar-Charts are diagrams which present graphically the division of a whole into its component parts (Fig. 29).

For instance, departmental expenses by causes can be shown by means of a component bar-chart; as can the amount contributed by each of several departments to total sales, or the comparative data showing proceeds and costs of sales.

Percentage Distribution Diagrams, showing the distribution of the components of a whole upon a percentage basis usually take the form of bar-charts or pie- (or circular-) charts.

The figures must first be calculated as a percentage of the whole, which is represented on the scale as 100.

Sometimes pie-chart calculations are made upon the basis that there are 360 degrees in a circle, but it is much more convenient to divide the circumference into 100 equal parts (as in Fig. 31) in the same way as a bar diagram. In Fig. 30 the data used for Fig. 29 have been converted into percentage form.

In Fig. 31 a pie-chart is illustrated. It is easier to read the percentage on such a chart when the circumference is divided as shown, instead of with 360°. It should be noted, however, that for comparison purposes bar-charts are to be preferred, as it is easier to compare the length of the bars and the component parts, than to compare two circles of different size. In the latter, *percentages* can be compared with equal facility, but *not*

Production of Castings. Proceeds and Costs per Ton.	January.	Last Year.	Proceeds and Costs shown as Percentages. This Year.	Last Year.
Realised per ton	£380	£300	100%	100%
Cost—				
Metal	80	90	21·0	30·0
Moulding, etc.	180	200	47·3	66·7
General Expenses	30	50	8·0	16·6
Total	290	340	76·3	113·3
Profit or Loss per Ton	+ 90	− 40	+ 23·7%	− 13·3%

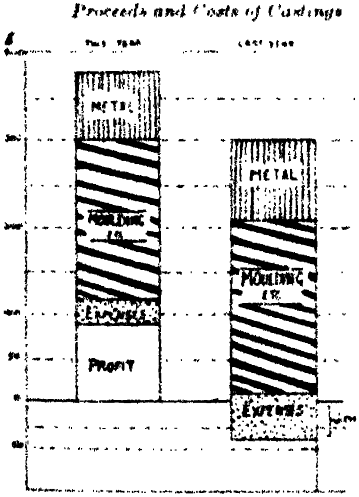


FIG. 29.—COMPONENT BAR-CHART

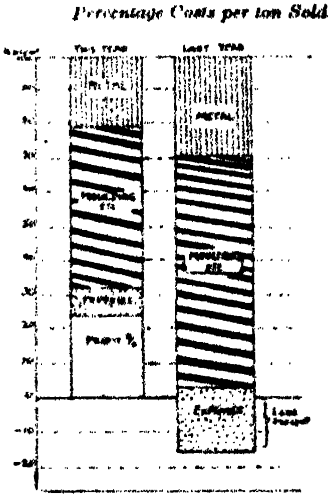


FIG. 30.—COMPONENT BAR-CHART (PERCENTAGE BASIS).

the aggregate whole owing to the difficulty of comparing circular areas. It is difficult to compare most diagrams in which area has to be relied upon, and excepting for giving approximate general impressions they should be avoided. Hence, even on bar-charts the bars should be of equal width, so that the eye is concerned only with length.

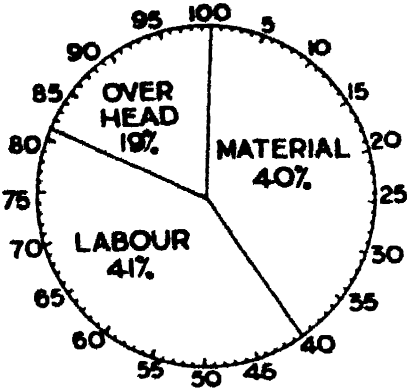


FIG. 31.—PIE-CHART FOR PERCENTAGE COMPONENT PARTS. THE SECTORS MAY BE COLOURED.

QUESTIONS

1. Mention, and briefly describe, three ways of presenting statistics diagrammatically.

2. What methods may be employed to present diagramatically, (*a*) Density of distribution territorially; (*b*) Comparative sizes? Give one illustration of each of (*a*) and (*b*).

3. Write notes on the following: (*a*) Simple Bar-Charts, (*b*) Compound Bar-Charts, (*c*) Circle- or Pie-Charts.

4. What is a Progress Chart? Illustrate.

5. Describe two forms of percentage distribution diagrams.

CHAPTER X

GRAPHIC METHODS (*contd.*)

II. CURVE CHARTS OR GRAPHS

IT must be observed that although much used for general purposes, diagrammatic methods are less effective statistically than the graphic method in which lines or curves to represent the movement of data are drawn to an appropriate scale.

The Construction of a Graph depends upon two straight lines intersecting at right angles to each other. These are known as co-ordinate axes. (See Fig. 32.)

The horizontal line is called the *x*-axis or the axis of abscissæ.

The vertical line is called the *y*-axis, or the axis of ordinates.

The point where they intersect is called the origin, and is designated 0 or zero point of the graph. These axes should be ruled to stand out distinctly.

Usually this framework of the graph is drawn on squared paper, *i.e.* paper ruled with feint horizontal and vertical lines intersecting at right angles. If not, it is usual to draw a sufficient number of fine or dotted lines to help the eye to locate any given point.

The Scale.—A decision is made that a certain distance on each of the co-ordinate axes shall represent a certain unit of measurement, much in the same way that a scale on a map is used. The scale can be varied to suit the requirements, namely, (*a*) To fit the paper, (*b*) To include all observations, (*c*) To avoid too sharp or too flat a curve. A single square of $\frac{1}{8}$ inch or other distance may be used to represent one shilling, a pound, £100, £1000 or other sum; or any other numerical item such as the number of employees, quantities of things bought or sold, etc.

The exact position of any point can be located by measuring from the *x*-axis and from the *y*-axis.

Distances measured upwards from the *x*-axis are positive, those downwards or below *x*-axis are negative.

Any convenient scale may be used, and the scales for the

x-axis and y-axis need not be the same. The scale should be clearly marked, and as far as possible should consist of whole numbers.

In Fig. 32 *ox* is the abscissa.
oy is the ordinate.

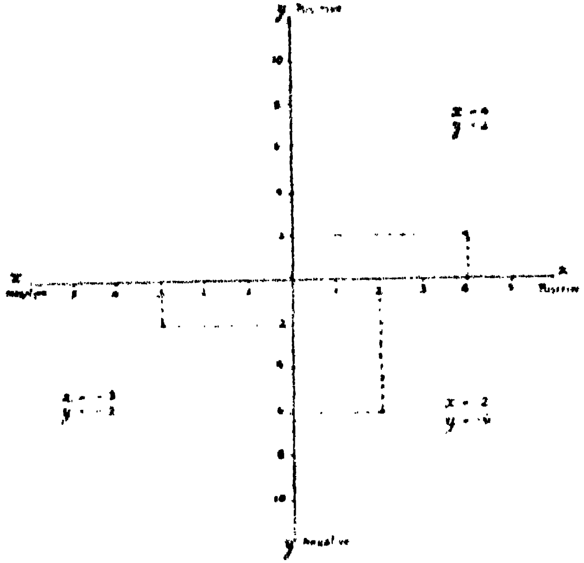


FIG. 32.—THE CONSTRUCTION OF GRAPHS.

The procedure may be best explained by the aid of an example (Fig. 33) :—

Suppose the daily output of articles for the first four days of the month were, 1st day, 100 ; 2nd, 300 ; 3rd, 600 ; 4th, 500.

> Let the axis *ox* represent days, each division on it being one day ; and let the axis *oy* represent units of output, each division on it being 100 units.

Then a vertical dotted line drawn upwards from point 1 on the x-axis, and a horizontal dotted line drawn to the right from *oy* at point 1 on *oy* will intersect. These intersecting lines are co-ordinates. Continue the procedure for each of the four days, so that four points of intersection are found. Connect each of these four intersection points by a line, as shown in Fig. 33. This line is called " a curve."

The curve gives an immediate impression as to how many units have been made each day.

Instead of connecting each point, a line or bar could have been dropped to the axis *ox*, but it will be seen that the curve in Fig. 33 representing the bars gives a view of the change more clearly than the bars.

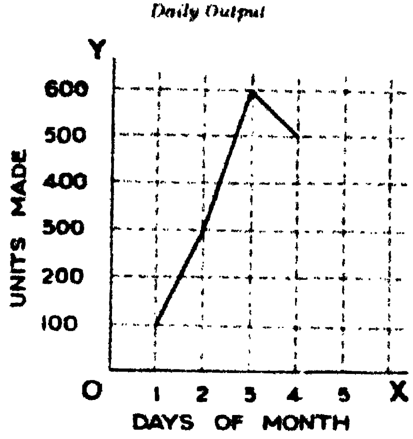


FIG. 33—SIMPLE EXAMPLE OF A "CURVE."

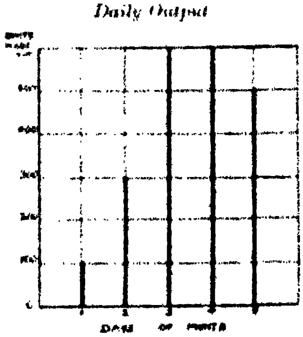


FIG. 34.

Lines dropped from points plotted are not as effective as the curve in Fig. 33.

Using the Line Chart.—As the distances between the horizontal lines on the chart are all equal in size, all similar increases, or decreases, in figures plotted are represented by the same vertical distance on it. If, therefore, two or more comparable sets of figures are plotted on squared paper, the vertical rise and fall of the curve indicates the magnitude of each series plotted in terms of actual quantities. The graph is known as an arithmetic or natural scale graph, to distinguish it from ratio graphs in which the spacings are graduated on a logarithmic scale such as is used on a slide rule. Logarithmic scales will be dealt with later.

Historigram.—When a chart of the type shown in Fig. 33 relates to data in a time series it is called a Historigram. This term should not be confused with "Histogram," a different type of graph which is illustrated in Fig. 56.

Selecting the Scale and Grid.—It is necessary to plan out what space will be available both vertically and horizontally to accommodate the number of items to be plotted on the graph. The manner in which the graph is to be used will determine what size the paper should be, *i.e.* whether to be consulted at a table by an individual, or on a wall by one or more people. The size determined a scale must be adopted which will enable all the desired data to be shown. Usually the base line is made a little heavier for emphasis.

If squared (or co-ordinate paper as it is called) is used, each square can be assigned a scale value. If plain paper has to be used, the scale distances will have to be marked off on the abscissa and ordinate axes, always remembering that the point of intersection of these co-ordinate axes must be zero. If any of the data is negative, space must be allowed for the curve to lie to the right of the ordinate or below the abscissa.

Equal distances anywhere on the horizontal scale must represent equal values; similarly must any equal distances on the vertical scale.

The point of intersection of the co-ordinate axes must be zero in a natural scale graph, and generally the abscissa axis should be the base line. This should be regarded as a fixed rule, although exceptionally for some purposes, instead of the zero line, a line of principal reference may be used, *e.g.* a line representing, say 100 per cent., an average or a normal with which the plotted curve has to be compared. When drawing a graph representing a series of data over a period of time the *zero line should always appear*. This does not apply to a ratio or semi-logarithmic chart, as described in the next chapter.

Deciding whether Scale should be on Abscissa or Ordinate Axis.—In time charts the horizontal base should be used for the time scale, *e.g.* years, months, days, etc.

An Independent Variable (*e.g.* days of the month in Fig. 33) is usually plotted on the horizontal axis *ox*.

A Dependent Variable (*e.g.* units made, in Fig. 33) is plotted on the vertical axis *oy*. Thus days of the month form the basis upon which the information of units made is studied. A caption explaining the units should appear, preferably at the top point of the vertical scale.

Plotting the Curve.—As the base line is where the independent variable is marked off, and readings will be made from those points, it is only necessary to find the position where each ordinate intersects a vertical line from the corresponding point on the base line. Usually the point plotted is placed on the vertical line run up from the base markings, but some prefer to plot the point *between* the lines. (See Figs. 33 and 35). Whichever method is used the designation (year, month, etc.) should be placed exactly below the position of the point.

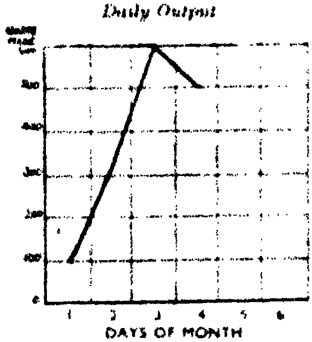


FIG. 35.—GRAPH SHOWING POINTS PLOTTED BETWEEN THE VERTICAL LINES FROM THE ABSCISSA.

Connecting the Points on the Chart.—When connecting the points the curve should be emphasised by making it heavier than the ordinates, or background lines and squares. Greater emphasis may sometimes be given by blocking in the areas between the curve and the base or reference line as in Fig. 36.

The method of connecting varies :—

(*a*) For non-cumulative figures the points should be connected by straight lines. The line simply indicates the direct movement from one point to the next (Fig. 33).

(*b*) For cumulative totals or a continuous series the line connecting should be in as smooth a curve as possible, avoiding angles. The curve passes through every point as in (*a*), but smoothly curves from one to the other as though the data were accumulating gradually and evenly (Fig. 57).

This method of drawing the curve must not be confused with what is known as "smoothing" curves, described in the next paragraph.

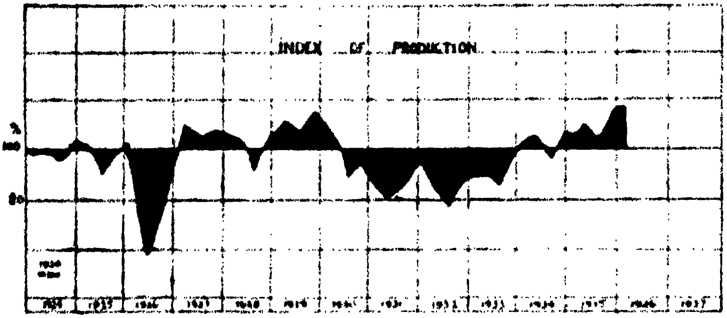


FIG. 30.—SHOWING EMPHASIS ON FLUCTUATIONS BY BLOCKING IN BETWEEN THE CURVE AND A LINE OF REFERENCE.

Smoothing Curves.—When there are frequent fluctuations up and down in a curve there is often a broad general movement or trend of the whole curve, quite apart from these frequent irregularities in the line. For the purpose of indicating this general trend more clearly methods of smoothing out the irregularities are employed, viz.:—

(*a*) A free-hand curve is drawn by judgment to indicate the main trend of the movement of the data illustrated, as in Figs. 57 and 99.

(*b*) The data are smoothed by a series of averages or a series of totals, and these computed figures are plotted instead of the original actual figures (Fig. 37).

(*c*) Mathematical calculations, such as the method of algebraic least squares, are made, from which a smoothed curve may be plotted, but as these are rarely utilised in business, they will be described in a later chapter (see page 147).

Two or More Series of Data may be shown on the same graph, but care must be taken to see that the curves are distinct and easily followed without confusion.

When two or more variables are plotted on the same graph we may use :—

(*a*) Different coloured lines.

(*b*) Lines differently characterised, *e.g.* continuous, thin, thick, dotted, crossed, etc.

(*c*) Simple curves marked with a caption or designation which indicates the nature of the data represented by each curve.

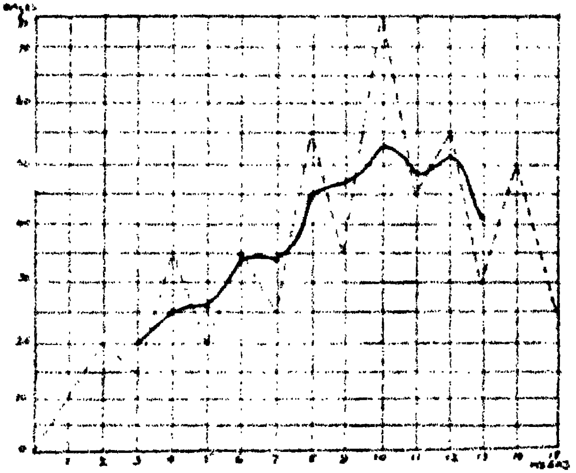


FIG. 37.—CURVE OF ACTUAL SALES (DOTTED CURVE) SMOOTHED BY USING 5-WEEKS MOVING AVERAGE (SOLID CURVE). FOR METHOD OF FINDING THIS AVERAGE SEE PAGES 84 AND 150.

In the case of (*a*) and (*b*) a "key" or "legend" explaining what each represents should be inserted beside the chart or may be shown on a convenient space on the chart.

In Fig. 41 both method (*b*) and (*c*) are employed together.

Title and Notes.—The title should usually be above the chart, outside the chart itself. Explanation and footnotes are usually placed below the chart, but are sometimes shown in a panel drawn on an open part of the chart away from the curves (Fig. 21).

In business it is not usually necessary to show the data which the graph represents, but sometimes it is required, in which case it may be shown in the following ways :—

(*a*) On a separate sheet.
(*b*) In a panel beside the graph.
(*c*) In a panel in a space on the graph itself.
(*d*) Opposite the ends of the abscissæ and ordinates.
(*e*) Against the plotted points.

In the case of (*d*) and (*e*) there is a danger of losing clearness through too much crowding of information. There is no hard-and-fast rule or practice, and the choice must be dictated by convenience and clearness.

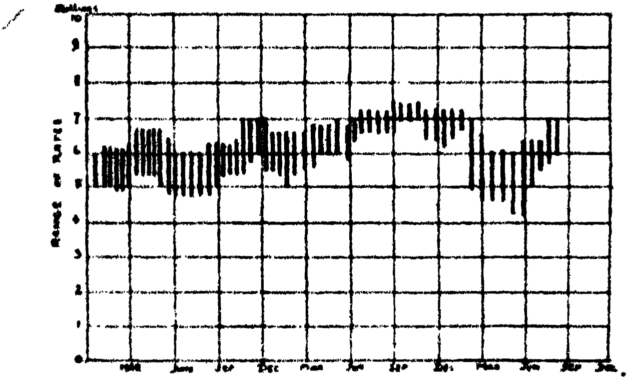


FIG. 38.—A ZONE CURVE SHOWING RANGE OF MAXIMA AND MINIMA OF A SERIES OF BONUS RATES.

Zone and Band Curves.—Instead of plotting a simple curve or curves, an effective representation of the maxima and minima of a series of data can be obtained by plotting a curve for the maxima and one for the minima and then filling in the space between the two with vertical lines or shading (Fig. 38). This is a zone curve.

A zone curve could be used to show earnings, rates of pay, prices of goods, interest rates, number of sales per person, output by men or shops over a period, etc.

In band curves, the constituent parts of a whole (*e.g.* components of cost and selling prices of articles) are plotted one above the other, thus creating a series of bands or zones each of which can be shaded in so that the whole space between the top curve and the base line is filled in solidly (Fig. 39). If the data are put in percentage form the whole chart may depict 100 per cent. and the bands shaded in the percentage each component represents of the whole.

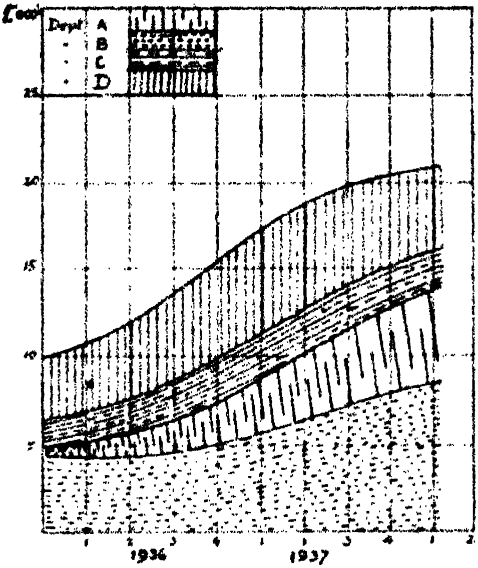


Fig. 39.—A Band Curve Chart Showing Composition of Total Sales by Departments.

The Use of Arithmetic Charts in their various forms, and particularly as applicable for the presentation of business statistics, is considered further in Chapter XXI (p. 183).

Questions

1. What is meant by "historigram"?

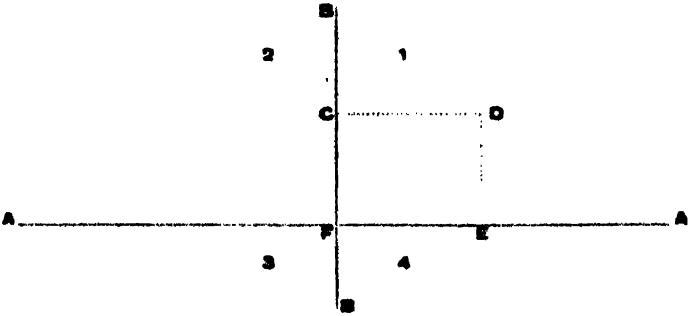
2. On what scale should the independent variable be plotted?

3. What is a smoothing curve? How may such a curve be decided?

4. What kind of chart do you consider best for presenting several series of data over a given period? How would you distinguish each series?

5. What distinguishes the arithmetic and logarithmic scales?

6. State five general rules which should be borne in mind in the construction of graphs. (*London Association of Certified Accountants.*)



From the above diagram, which shows the construction of a graph, ascertain the following:—(*a*) The "Axes of Reference." (*b*) The abscissa of the point D. (*c*) The ordinate of the point D. (*d*) The co-ordinate. (*e*) The Quadrants. (*London Association of Certified Accountants.*)

7. State the general rules for the construction of graphs. (*Incorporated Accountants* (*Final*).)

8. Draw a graph illustrating the following figures:—

Persons Liable to Super-tax for the Year Ended 5th April, 1927, Classified by the Amount of their Income.

Income (£).	Persons.	Income (£).	Persons.	Income (£).	Persons.
2,000–2,500	22,724	7,000–8,000	2,970	30,000–40,000	586
2,500–3,000	15,698	8,000–10,000	4,130	40,000–50,000	278
3,000–4,000	18,646	10,000–15,000	4,603	50,000–75,000	299
4,000–5,000	10,383	15,000–20,000	1,821	75,000–100,000	97
5,000–6,000	6,473	20,000–25,000	920	Over 100,000	147
6,000–7,000	4,362	25,000–30,000	559		
				Total	94,676

(*N.A.L.G.O.*)

CHAPTER XI

GRAPHIC METHODS (*contd.*)

III. GRAPHS SHOWING RATES OF CHANGE

THE graphs described so far have been constructed on the natural scale, and although these show admirably the actual extent of absolute increases or decreases, they do not show the rate of change, *i.e.* relative movements.

This may be explained by a simple example :—

	Sales.	Annual Increase.	Percentage Increase.
1935	£1000		..
1936	2000	£1000	100
1937	3000	1000	50

These sales figures plotted on an ordinary natural scale graph would show the same vertical distance between the points plotted. The same distance would also be shown if a sales total increased from £100,000 *to* £101 000 (an increase of £1000, but of only 1 per cent.). If instead a ratio scale is used, the rate of increase is clearly shown.

Ratio-Scale Graphs must be used whenever it is desired to show relative movements. On such graphs equal vertical distances represent equal *proportionate* movements.

On natural-scale graphs equal vertical distances represent equal *absolute* movements.

A ratio-scale graph may be constructed in two ways :—

(1) By plotting the logarithms of the numbers on ordinary co-ordinate paper.

(2) By plotting the actual numbers on ratio-ruled paper, *i.e.* the distances between the horizontal lines are scaled logarithmically.

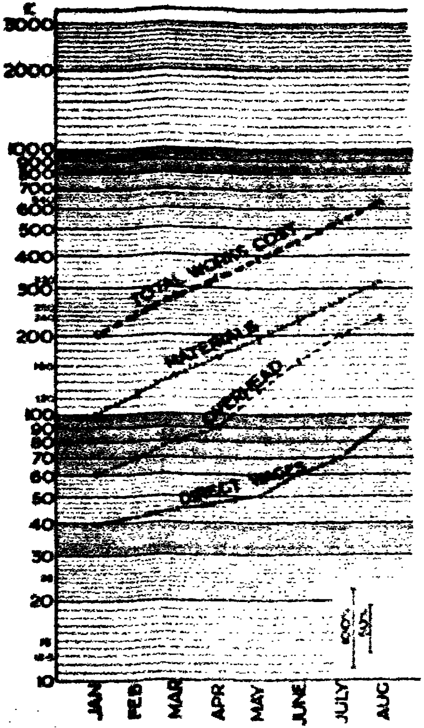


FIG. 40.—Semi-Logarithmic Chart Ratio-Ruled Paper.

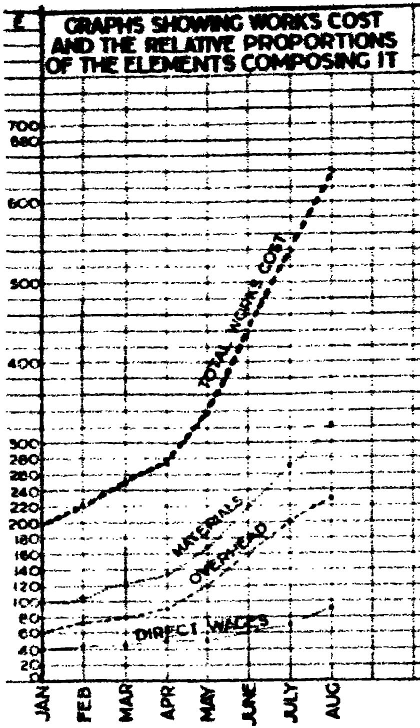


FIG. 41.—Natural or Ordinary Scale. Natural-Scale Graph Compared with

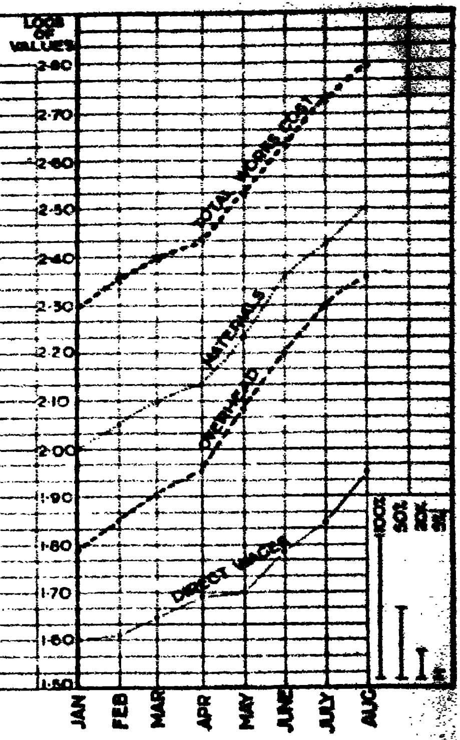


FIG. 42.—Logarithmic Scale on Ordinary Scale Ruling.

As it is the vertical scale only that is scaled on a logarithmic basis, these graphs are often termed **Semi-logarithmic Graphs.** Both Fig. 40 and Fig. 42 are examples; also Figs. 43 and 44.

As the use of logarithms is not popular nor well understood by most business people, it is probably better to use ratio-ruled paper, like that shown in Fig. 40.

A ratio-ruled graph is preferable for other reasons—namely: (*a*) The actual values may be shown at points on the vertical axis, so that both the actual changes can be read from the absolute figures shown on the scale, and the relative changes from the curves drawn, and (*b*) certain series of figures which would not otherwise be shown graphically can be represented, viz. an extensive series showing a great range of variation.

In the ratio-ruled paper the vertical scale is measured off logarithmically, so that each distance from 1 to 10, 10 to 100, 100 to 1000, etc. form what are known as cycles. The advantage is that when plotting figures differing widely in magnitude the well defined cycles can be used, and proportional changes are readily seen.

Summary of Features of the Ratio Scale.

(1) Equal vertical distances represent equal proportional changes. In Fig. 40 the distances between the points 20 and 40; 30 and 60; 200 and 400; 10 and 20 are all equal because they represent an increase of 100 per cent.

(2) Curves of equal slope, *e.g.* two parallel lines like the top two in Fig. 40, represent equal proportionate rates of change for the period January to August.

A curve steeper at one part means that the steeper part is changing at a more rapid rate.

(3) Zero and negative values cannot be shown. A logarithmic scale has no zero point.

(4) No base line is necessary, and the whole graph, or different curves on it, may be moved up and down without changing its meaning. The area under a ratio curve, so important in a natural-scale graph, is meaningless on a ratio chart.

This is an extremely convenient advantage in that two curves widely separated on a ratio chart may be brought close together for greater ease of comparison. The two curves in Fig. 43 have been brought together in Fig. 44, thus making the comparison better.

(5) A series showing a great range of variation can be plotted very conveniently. An extensive range of wide disparity which could not possibly be plotted on a natural-scale graph, can be admirably represented on a ratio chart.

(6) It is not suitable for showing an analysis of a whole into its constituent parts.

(7) Contrary to the case of the arithmetic chart, two or more scales may be used on a single band of the ratio-ruled paper (Fig. 44).

(8) Two or more series of entirely different units, *e.g.* tons and gallons or ounces, can be compared on the same graph to show rates of change. This cannot be done on a natural-scale chart.

(9) A natural-scale graph may be converted into a ratio-scale graph by plotting logarithms of the values on the vertical scale; or by plotting the values on semi-logarithmic graph paper.

Fluctuations in Sales of Fowls and Eggs compared relatively

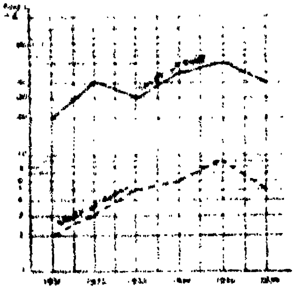


FIG. 43.—RATIO-SCALE GRAPH COMPARING SALES OF TWO PRODUCTS.

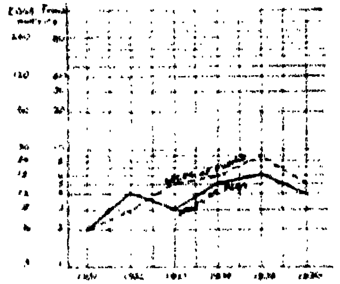


FIG. 44.—RATIO CHART COMPARISON FACILITATED BY BRINGING CURVES CLOSER TOGETHER.

The Uses of the Ratio Graph.—The statistical facts relating to the following business matters may be shown advantageously on a ratio graph :

Sales and materials purchased or produced.
Production output and costs.
Number of employees.
Labour losses and absences.
Wages and expenses.
Revenues.
Net profits.
Gross profits.

QUESTIONS

1. What is a ratio-scale graph? What purpose does it serve?

2. What are the special features of logarithmic or ratio charts? Suggest types of data which could advantageously be presented graphically in this form. (*N.A.L.G.O.*)

3. State the chief advantages of a logarithmic scale graph as compared with a natural scale graph. (*London Association of Certified Accountants.*)

4. Chart the figures of the population of Great Britain and Ireland given below so as to stress the declining rate of increase of the population over the last hundred years.

Population of Great Britain and Ireland (in thousands)

1821	20,894	1881	34,885
1831	24,029	1891	37,733
1841	26,731	1901	41,459
1851	27,391	1911	45,222
1861	28,927	1921	47,123
1871	31,485	1931	49,003

(*Union of Lancashire and Cheshire Institutes.*)

5. The following table gives the fluctuations in price of three commodities, extending over the period as indicated.

Year.	Average price of commodity A during year.	Average price of commodity B during year.	Average price of commodity C during year.
1925	£40 per ton.	3*d.* each.	5*s.* per ounce.
1926	£37 ,,	2¾*d.* ,,	4*s.* 9*d.* ,,
1927	£39 ,,	2½*d.* ,,	4*s.* 6*d.* ,,
1928	£42 ,,	2½*d.* ,,	4*s.* 6*d.* ,,
1929	£35 ,,	2⅝*d.* ,,	4*s.* 9*d.* ,,
1930	£37 ,,	2¼*d.* ,,	4*s.* ,,
1931	£35 ,,	2¼*d.* ,,	4*s.* 3*d.* ,,
1932	£31 ,,	1¾*d.* ,,	4*s.* 3*d.* ,,

You are required to submit an historigram showing the comparative price changes in these three commodities. (*London Association of Certified Accountants.*)

6. State the characteristics of: (*a*) Natural scale graphs; and (*b*) Ratio-scale graphs. How may the former be converted into the latter? (*Incorporated Accountants (Final).*)

7. Graph the following figures on a Ratio or Logarithmic Chart, so as to bring out comparatively the seasonal fluctuation present.

Number of Insured Males Unemployed (Thousands) in U.K., 1936.

	Jan.	Feb.	Mar.	Apr.	May.	June.
Coalmining . . .	183	189	203	218	216	273
Shipbuilding . . .	57	56	55	52	51	47

	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Coalmining . . .	233	189	191	176	161	146
Shipbuilding . . .	48	49	48	45	44	43

(*N.A.L.G.O.*)

8. On 1st January, 1921, a certain city A contained 100,000 houses, and 10,000 additional houses were built there each year from 1921 to 1930 inclusive. In another city B, which had also 100,000 houses on 1st January, 1921, the number was increased at a constant annual rate of 7·1773 per cent. Draw a graph on an ordinary arithmetic scale illustrating the annual housing statistics of these two cities from 1921 to 1930; and draw another graph on a logarithmic scale illustrating the same statistics. Which is the more useful graph? (*N.A.L.G.O.*)

CHAPTER XII

ACCURACY AND APPROXIMATION

The Value of Approximations.—The accounts of a business are expected to be absolutely accurate, and in certain statistical reports the same degree of accuracy is demanded; but in many forms of statistical work much approximation and estimation can be made without loss of importance—in fact such methods may result in greater clarity.

Very commonly, for instance, shillings and pence, 10's or 100's, may be disregarded, and for some purposes even greater aproximation is made. The important feature is the purpose for which the data will be required. In statistical sales reports approximate figures correct to the nearest £10 or £100 will often suffice for a small business, but for a larger concern figures to the nearest £500 or £1000 may be equally satisfactory.

Many investigations can be carried out with reliance on estimates, and results based on approximations may often be appreciated more easily and clearly. Conclusions arrived at from such statistics will often be exactly the same as they would have been had precise figures been utilised.

How Degree of Accuracy may be Stated.—The degree of accuracy should usually be stated, and the following are examples of how this may be indicated, using for illustration a production output of pig-iron of 1000 tons, which weight is known to be accurate within 10 cwt. under or over. The position may be stated in any of the following ways:—

(1) The output is 1000 tons in round numbers.

(2) The output is between 999½ and 1000½ tons or 999·5 and 1000·5 tons.

(3) The output is 1000 tons, plus or minus a difference not more than half a ton. This may be written statistically as 1000 ± 0·5 ton.

(4) The output is 1000 tons correct to 0·5 per mille, or 0·05 per cent.

For very many purposes large numbers of figures can be stated advantageously in round numbers, even when actual figures are known.

Methods of Approximation.

(1) *To the Nearest Whole Number.*—Particulars of comparatively large numbers, such as the production of pig-iron or coal in various districts, may be sufficiently accurate if stated to the nearest 1000 tons.

> Then 869,390 tons would be expressed as 869,000 tons to the nearest thousand; or 869,700 tons as 870,000 tons to the nearest thousand.

Again, the ratio of working expenses to gross receipts expressed as a percentage calculated as 64·42867 per cent. would be sufficiently stated for most purposes as 64 per cent., or with slightly closer accuracy as 64·43 per cent.

(2) *Approximation by Using the Next Highest Whole Number, e.g.*

869,390 tons would become 870,000,
869,700 tons would also become 870,000,
and 64·42867 per cent. would become 65 per cent.

(3) *Approximation by Discarding Digits* to arrive at a round number, thus

869,390 tons becomes 869,000,
869,700 tons becomes 869,000,
and 64·42867 per cent. becomes 64 per cent.

Care is necessary if approximated figures are used for calculations involving multiplication, division or extraction of a root, because the error in the figures themselves becomes multiplied or divided and may be greatly misleading.

Approximations of large numbers cause little or no variation in percentages based upon them.

Various degrees of approximation and the effect on the percentages are shown in the tables in Fig. 45, from which it will be seen how little difference results.

Error in Approximation.—The differences between actual and approximated figures are known as "error." This does not mean a mistake, but is merely a statistical term for such a margin of difference between an actual figure and an approximation of it.

It may be noted here that statistical errors may arise in various ways, the chief of which are :—

(*a*) Errors of origin due to insufficient definition of the information required, biassed collection of data, and erratic features inherent in the data themselves.

Receipts.	Gross Profit.	Percentage of Profit to Receipts (correct to second place of decimals).	Percentage correct to the nearest 1 per cent.
1. *Table Showing Gross Receipts and Profits for Three Years*			
49,461,428	12,365,357	25·00	25
48,096,472	12,711,898	26·43	26
53,960,249	15,346,295	28·44	28
2. *Same Data Approximated to Nearest Thousand.*			
Note.—The approximation has not affected the percentages.			
49,461	12,365	25·00	25
48,096	12,712	26·43	26
53,960	15,346	28·44	28
3. *Same Data Approximated to Nearest Hundred Thousand.*			
Note. There is still no appreciable difference in the percentage.			
495	124	25·05	25
481	127	26·40	26
540	153	28·33	28

FIG. 45.—EXACT FIGURES IN THE FIRST SECTION APPROXIMATED IN SECTIONS 2 AND 3 TO SHOW HOW LITTLE PERCENTAGES ARE AFFECTED.

(*b*) Errors involuntarily made in counting, measuring, calculating, describing and approximating.

(*c*) Errors due to utilising too limited a number of items or samples.

Statistical errors may be measured (*a*) absolutely, or (*b*) relatively.

(*a*) **Absolute Error.**—If the actual figure of sales is say £9900, and the estimate or approximated value is £10,000, there is a difference or error of £100. This is the Absolute Error.

(*b*) **Relative Error.**—If the absolute error of £100, above, is shown as a ratio of the approximated figure, we have $\frac{100}{10{,}000}$, *i.e.* 0·01, and this is called the Relative Error.

If expressed as a percentage error this error would be 1 per cent. of the approximated figure, calculated thus:

$$\frac{100}{10{,}000} \times 100, \text{ i.e. } 1\%$$

Errors may be Biased or Unbiased, compensating or uncompensating. When numbers are approximated to the nearest ten, hundred, thousand, etc., some of the approximated figures will be higher and some lower than the actual numbers.

Providing there are many numbers so dealt with, the differences will tend to compensate one another (*i.e.* cancel out), and the errors are said to be **Unbiased or Compensating.**

When, however, all the numbers are approximated to the nearest ten, hundred, etc., below, or all to the nearest above the actual number the error is **Biased** or **Cumulative,** and the more numbers there are the greater will the total error be.

Table of Biased and Unbiased Errors.

Exact Numbers. *a*.	Correct to nearest 1000. *b*.	Absolute error Unbiased. *c*.		Correct to next 1000 over. *d*.	Absolute error Biased. *e*.
49,362	49	− 362		50	638
48,096	48	− 96		49	904
53,760	54		+ 240	54	240
61,428	61	− 428		62	572
50,460	50	− 460		51	540
96,672	97		+ 328	97	328
60,249	60	− 249		61	751
60,503	61		+ 497	61	497
50,240	50	− 240		51	760
530,770	530	− 1835	+ 1065	536	5230
		− 770			

FIG. 46.—COMPARISON OF BIASED AND UNBIASED APPROXIMATIONS.

The absolute error in the total (Fig. 46, column c) is only − 770 in a total of 530,000.

The relative error of the total of unbiased items (column *c*) is $\frac{770}{530{,}000} = 0{\cdot}001453$, and is so small as to be negligible.

The relative error of the total biased error (column *e*) is $\frac{5,230}{536,000} = 0{\cdot}00975$.

The absolute error of the total in column (*e*) is biased, and it will be noticed that the size of the error is important.

When approximating figures in business statistics, this important difference between biased and unbiased approximations should be borne in mind, and usually unbiased approximations should be used.

Estimating Unbiased Absolute Error in Approximations.—Although it may not be of great service in business, it is useful to know that when the actual figures which have been approximated are not known an estimate of the absolute error in a total of a number of items approximated in a known unbiased manner may be found as follows :

$$\left.\begin{array}{r}\text{Average Absolute}\\ \text{Error of the items}\end{array}\right\} \times \sqrt{\text{Number of items}}.$$

In the example above, as the approximation is to the nearest 1000, the absolute error would most likely be the average of 0 to 499, which is about £249·5, or say £250 and there are 9 items, hence the unbiased error would probably be

$$250 \,.\, \sqrt{9} = \pm\ £750$$

which compares with − £770 in the figures used. The result does not always come as close as this, and the formula gives only a rough estimate.

To Estimate the Relative Error of a total of a number of items affected by the same unbiased error, find the ratio between the estimated absolute error (*e.g.* £750 above) and the approximate total (*e.g.* £530,000) equivalent to the following :—

$$\frac{250 \times \sqrt{9}}{530,000} = \frac{750}{530,000} = 0{\cdot}00115.$$

This compares with 0·00145 calculated on the known numbers in Fig. 46.

To Estimate the Biased Absolute Error in a total of approximated figures. It is apparent that the minimum error is 1 × No. of items, and using the figures in the example above, the maximum can be 999 × No. of items ; but the most likely error would be : Average Absolute Error × No. of items or 499·5 × 9 = 4495·5, which compares with the known error £5230 (in Fig. 46, column *e*).

Approximation in Multiplication.—The degree of accuracy in multiplying approximated figures is sometimes required. The following is an example :

108 × *112*00, the accurate figures are in italics.

To find the accuracy of the result it is convenient to use an algebraic solution.

Let A be the multiplier and B the multiplicand,
x be the possible error of the multipliers, and
y the possible error of the multiplicand.

The product will be $(A \pm x)(B \pm y)$ which with relatively negligible values of x and y may be written $AB \pm (Bx + Ay)$. Applying this to the figures above the possible error in 108 is ± 0·5 and in 11,200 is ± 50, hence we have :

$$
\begin{aligned}
& 11{,}200 \times 108 \pm (108 \times 50 + 11{,}200 \times 0{\cdot}5) \\
= \; & 1{,}209{,}600 \pm (5{,}400 + 5{,}600) \\
& 1{,}209{,}600 \pm 11{,}000
\end{aligned}
$$

The Use of Percentages.—Percentages must be used with caution. Many mistakes arise through incorrect observance of the basis of percentage calculations, and for this reason, among others, that ratios are preferred, particularly for comparative purposes. When percentages are used for comparisons it is important to ascertain that the data are homogeneous and capable of being compared, as, if they are not, then the percentages are not comparable.

But little information is afforded when gross profits are stated to be, say, 36 per cent., or net profits 7½ per cent. It is necessary to know the amount of turnover or capital involved.

A common type of question arises regarding variations in percentages concerning wages and prices, *e.g.*, wages were reduced 5 per cent., raised 10 per cent., lowered 20 per cent., and then raised 25 per cent. over a stated period of time. Ascertain the change in wages over the whole period.

There are two alternative replies, according to whether the basis is (*a*) the original wages or (*b*) the rate of wages paid at the date changes were made. The calculations in the first supposition are as follows (using 100 as the original wages for convenience) :—

5 per cent. reduction ∴ Wages became $\frac{95}{100} \times 100 = 95$

10 per cent. increase ∴ ,, ,, $95 + \frac{10}{100} \times 100 = 105$

20 per cent. decrease ∴ ,, ,, $105 - \frac{20}{100} \times 100 = 85$

25 per cent. increase ∴ ,, ,, $85 + \frac{25}{100} \times 100 = 110$

i.e. the final result is that over the period wages were increased 10 per cent. on the original wages.

Then, using the second supposition :—

5 per cent. reduction ∴ wages became $\frac{95}{100} \times 100 = 95$

10 per cent. increase ∴ ,, ,, $\frac{110}{100} \times 95 = 104{\cdot}5$

20 per cent. decrease ∴ ,, ,, $\frac{80}{100} \times 104{\cdot}5 = 83{\cdot}6$

25 per cent. increase ∴ ,, ,, $\frac{125}{100} \times 83{\cdot}6 = 104{\cdot}5$

or, the result can be directly calculated thus :—

$$\frac{95}{100} \times \frac{110}{100} \times \frac{80}{100} \times \frac{125}{100} \times 100 = 104{\cdot}5$$

i.e. the final result is that over the period wages were increased 4·5 per cent. on the original wages (as compared with 10 per cent. calculated under the first supposition).

It is not possible to add, subtract or average percentages excepting when the basic factors upon which the percentages are based are comparable and the bases equal.

Questions

1. What do you understand by statistical "Error"? How far does it, in practice, affect the value of statistical investigations? (*London Chamber of Commerce.*)

2. How may approximations be made? What purpose do they serve?

3. Explain "absolute error," "relative error," "biased error."

4. Work out (showing your workings) the most absolutely accurate result of multiplying—*726* by *10200*. When the digits in italics only are accurate. (*Incorporated Accountants* (*Final*).)

5. The estimated expenses of an industry consist of £487,000 for wages and £216,000 for other expenses; and the estimated receipts

from sales are £750,000. Calculate the estimated net profits, giving the limits of possible error on the assumption that the amounts stated are liable to errors of 2 per cent., 8 per cent., and 9 per cent. respectively. (*Union of Lancashire and Cheshire Institutes.*)

6. Find the sum and difference of 542,936 and 135,706, both numbers being correct to 2 per cent.; and express in percentage form the degrees of accuracy of the results. (*Union of Lancashire and Cheshire Institutes.*)

7. The annual death rate in a certain provincial city is between 20 and 24 per thousand, and the population is estimated at 800,000 (within 7 per cent.). Calculate the number of deaths per annum, and show the degree of accuracy in your answer. (*London Association of Certified Accountants.*)

8. An estimate of the population of a town is given as 49,000, whereas actually the population is 50,000. Compute (*a*) the absolute error, (*b*) the relative error, and (*c*) the percentage error. (*London Association of Certified Accountants.*)

9. Determine the degree of accuracy in multiplying *648* × *21200*. The accurate figures are italicised. (*Incorporated Accountants* (*Final*).)

10. Twenty-nine sums (each exceeding 2000) are written to the nearest 1000 and totalled. The true aggregate total is 4,536,541. What is the best estimate of (*a*) The absolute error in the total; and (*b*) the relative error in the total? (*Incorporated Accountants* (*Final*).)

11. What degrees of accuracy would you deduce from the following entries in a table of measurements of length, viz.: centimetres 7, 7·0, 7·00. (*Incorporated Accountants* (*Final*).)

12. —". . . . the average advance in weekly full-time rates (of wages) is 64 per cent. at December, 1933, as compared with August, 1914 . . . then the cost-of-living index figure showed a rise of 42 per cent. as against the position at the outbreak of the Great War. The obvious conclusion which will be drawn is that there has been a substantial increase in real wages and that a considerable margin is available for cuts before the pre-war position is re-established."—From a leading article in the *Glasgow Herald* on "The Wage Level," 1/3/34.

Examine this piece of reasoning from the point of view of statistical methods. (*Corporation of Accountants.*)

13. Prove the two following propositions: (*a*) The error in an estimated sum is equal to the sum of the errors in the parts when each is multiplied by the ratio of the corresponding parts to the sum. (*b*) The error in the arithmetic average of several estimates is the sum of the errors of these estimates, when each is multiplied by the ratio of the corresponding estimate to that of the sum of the estimates. (*Corporation of Accountants.*)

14. In a certain industry wages were reduced 5 per cent., raised 10 per cent., reduced 15 per cent. and increased 20 per cent. over a protracted period, each percentage being calculated on the wages current when the change was effected. Ascertain the change in the whole period. (*London Association of Certified Accountants.*)

CHAPTER XIII

AVERAGES: I. ARITHMETIC AND GEOMETRIC

Statistical Groups.—The profit on some articles is large, and on others small; some wage-earners earn £1 a week, others more; the sales effected by one traveller are many, by others, few; the output of various piece-workers in a factory varies considerably as between those engaged in the work.

Different units of the same kind or species which exhibit variations are spoken of as **Variables.**

When the variations occur over a period of time, *i.e.* day by day, year by year or other periods, they are designated **historical variations.**

Ignoring time variations of the items in a group, two quantitative features can be noted:—

(*a*) The measurable characteristic, *e.g.* the weekly wage of workers in a factory; the output of pieceworkers producing similar articles.

(*b*) The frequency, *e.g.* the number of workers earning a given wage; the number of piece-workers producing a given quantity of similar units.

These are of great use in statistical arrangements, as will be described later.

Averages.—In a large group of variables of a like kind it is not easy to grasp the size of the items composing the group, but a typical specimen may be selected to represent the group. The choice of this typical example may be made in various ways, one of which is to find an average.

The Uses of Averages are:—

(*a*) To give a comprehensible concise idea of a group of items.

(*b*) To afford a basis of comparison with other groups.

If we are told the number of articles produced by each of 1000 piece-workers, we find it difficult to judge what is the usual output of the majority. When informed of the average, we have at once a definite idea. Again, given the production output of each the workers in two different factories producing similar articl it would be difficult to compare the respective efficiencies in th two factories; but if the average output of each factory is state a comparison can easily be made.

Kinds of Average.—There are four kinds of average (sometim called mean) which are commonly used :—

1. The Arithmetic Average or Arithmetic Mean.
2. The Geometric Average or Geometric Mean.
3. The Mode.
4. The Median.

The Arithmetic Average or Mean is the sum of a group o series of items divided by their number.

The simple arithmetic average is calculated thus: the number of orders taken by 10 salesmen on July 31st were: 9, 10, 5, 6, 9 12, 7, 8, 8, 6. The total orders were 80.

The simple arithmetic average is $\frac{80}{10} = 8$ orders per man.

The important point is that all the units must be homogeneous fo the purpose under consideration.

A Short Method of Averaging Large Numbers is as follows: Assume an average by inspection, find the deviation of number from this assumed average, take the average of the deviations, and add to or subtract from the assumed average. The result is the true average.

Example :—

Items.	Assumed Average.	Deviations.
853	850	+ 3
845	850	− 5
857	850	+ 7
862	850	+ 12
841	850	− 9
859	850	+ 9
861	850	+ 11
		Total + 28

Number of items = 7.
Average of deviations 28 ÷ 7 = 4.
True Simple Average 850 + 4 = 854.

FIG. 47.—SHORT METHOD OF AVERAGING LARGE NUMBERS.

Weighted Arithmetic Average.—In the simple average each item in the averaged group is regarded as of equal importance. Items vary in importance, however, and to obtain a representative average it is necessary to multiply each item by a suitable "weight" corresponding to its importance, and divide the total of the products by the sum of the weights. This is then called a weighted arithmetic average. The procedure is better shown by the following example.

Weekly Wages paid by P.Q.R., Ltd.

Weekly Wages. (*a*)	No. of Workers at each Rate of Wage. (*b*)	Product (*a*) × (*b*).
s.		
30	90	2,700
35	120	4,200
40	590	23,600
50	420	21,000
60	130	7,800
215	1350	59,300

$$\text{Weighted average} = \frac{59{,}300}{1350} = 44{\cdot}9s.$$

The simple average would be $\frac{215}{5}$ (total of column (*a*)) = 43*s.* per week, but the weighted average gives an average weekly wage of 44·9*s.* through taking account of the bigger proportion of workers at 40*s.* and 50*s.*

FIG. 48.—WEIGHTED ARITHMETIC AVERAGE.

Weighted Average using Approximate Weights.—When actual numbers of items can be used as weights, this is best, but for some purposes actual quantities are not known, then a close estimate may be made, and the estimated numbers can be used as weights. Estimated or approximated weights, if reasonably selected, make very little error in the resultant average. (See Fig. 49, page 81.)

It will be noticed that although the estimated figures differ considerably from the actual in Fig. 48, the average is 43·27*s.*, as compared with 44·9*s.* using actual numbers.

The following example shows the importance of using weighted values.

Coffee is sold at 1*s.*, 1*s.* 3*d.*, 2*s.*, 3*s.* and 4*s.* lb. The simple arithmetic average is :—

$$\frac{1s. + 1s.\ 3d. + 2s. + 3s. + 4s.}{5} = \frac{11s.\ 3d.}{5} = 2s.\ 3d.\ \text{lb.}$$

This figure is only of value if an equal weight of each grade were sold. In fact sales were 1362 lbs., 1200 lbs., 961 lbs., 450 lbs. and 222 lbs. These quantities must be used as weights :—

Weekly Wages paid in P.Q.R., Ltd.

Weekly Wages.	Estimated No. at Each Wage.	Weight.	Product.
s.			
30	150	1·5	45
35	150	1·5	52·5
40	600	6	240
50	400	4	200
60	150	1·5	90
		14·5	627·5

$$\text{Weighted average} = \frac{627\cdot 5}{14\cdot 5} = 43\cdot 27s.$$

FIG. 49.—WEIGHTED AVERAGE USING ESTIMATED WEIGHTS.

$$\frac{(1362 \times 1s.) + (1200 \times 1s.\ 3d.) + (961 \times 2s.) + (450 \times 3s.) + (222 \times 4s.)}{1362 + 1200 + 961 + 450 + 222}$$

$$= \frac{7022}{4195} = 1s.\ 8\cdot 09d.\ \text{per lb.}$$

The work can be reduced by using approximate relative weights :—

$$\frac{(14 \times 1s.) + (12 \times 1s.\ 3d.) + (10 \times 2s.) + (5 \times 3s.) + (2 \times 4s.)}{14 + 12 + 10 + 5 + 2} = \frac{72}{43} = 1s.\ 8d.$$

Advantages and Disadvantages of the Arithmetic Average.

Advantages.

(*a*) It is easy to understand and calculate, and is commonly used.

(*b*) It makes use of all the data in the group, and can be determined, therefore, with mathematical exactness.

(*c*) It can be determined when nothing more is known than the total value or quantity of the items, and the number of them, are known.

Disadvantages.

(*a*) It may give undue weight to, and be unduly influenced by, extreme abnormal items, *e.g.* in a business there may be a few earning very high wages, making the average higher than it would be without these included, and although the bulk of the workers are earning the same wage as those in another business, yet the average would give the impression, wrongly, that better wages are being paid in the first business.

(*b*) The average may be a value which does not correspond with a single item, *e.g.* (1) In *Figs.* 48 and 49, for instance, no worker earns exactly 44·8s. a week. (2) The average number of employees in each department of a business was found to be 4·86 persons, but obviously the actual number in each must be an exact whole number.

The arithmetic average is the most useful average for general purposes, and should be used unless particular reasons call for another method.

Uses of the Arithmetic Average in business include the following informative averages :—

Revenue and expenses.
Sales, daily, monthly, etc.
Output (weekly, etc.).
,, (per man, per machine, etc.).
Number of employees by sexes, per dept., etc.
Plant in service.
Departmental, etc. allocation of fixed expenses, *e.g.* rent by floor space.
Operation costs.
Sales per article.
Rates of pay.
Piecework earnings.
Bonus earnings.
Per capita consumption of articles.
Labour and machine hour rates for recovering oncost.

The Geometric Average or Geometric Mean.—The geometric average is used chiefly in averaging ratios, and in particular, index numbers, which express rates of change. It is useful because it does not give much weight to extreme items. This is a reason why it is used for the averaging of wholesale prices for the purposes of the Board of Trade Index of Wholesale Prices.

These ratios, which are usually termed " percentage relatives," should never be averaged by the arithmetic method, as the results would be false (unless all the quantities are equal).

Method of Computation.—The geometric average is the *n*th root of the product of *n* items in a group. For example :—

The geometric average 10, 100 and 1000 is $\sqrt[3]{10 \times 100 \times 1000} = \sqrt[3]{1,000,000} = 100$.

The arithmetic average is $\frac{10 + 100 + 1000}{3} = 370$.

The use of the geometric mean may be shown as follows :—

	1935.	1936.
The price of boxed peas is	2½*d.*	5*d.*
„ tinned beef is	20*d.*	10*d.*

These prices can be represented as follows, calling the price in 1935 100 :—

	1935.	1936.
Boxed peas . .	100	200 (*i.e.* double 1935).
Tinned beef . .	100	50 (*i.e.* half 1935).
Total . .	200	250
Arithmetic average	100	125
Geometric average $\sqrt[2]{100 \times 100}$	100	
„ „ $\sqrt[2]{200 \times 50}$		100.

The arithmetic average wrongly shows a percentage increase of 25 per cent., but the true fact, as shown by the geometric average, is that there has been no change in the average price condition of these two articles. This is because the price of peas was doubled, but the increase is offset by the halved price of meat (assuming equal weight to each).

When there are many items, *e.g* 49, the calculation of the 49th or *n*th root needs a simple knowledge of logarithms, and as their use also simplifies multiplication, a simple explanation of the use of logarithms for the purpose is given in Chapter XXII.

Advantages and Disadvantages of the Geometric Mean.

Advantages.

(*a*) It makes use of all the data in a group, and can be calculated with mathematical exactness, provided that all the quantities are greater than zero and positive.

(*b*) It is the only average that can be used to indicate *rate* of change, *e.g.* from 1935 to 1937 prices increased 5, 10 and 15·7 per cent. respectively.

What was the average annual increase ? It was not 10·2 per cent., as shown by the arithmetic average

$\frac{5 + 10 + 15{\cdot}7}{3} = 10{\cdot}2$, but 9·2 per cent., as shown by the geometric mean, viz. $\sqrt[3]{5 \times 10 \times 15{\cdot}7} = \sqrt[3]{785} = 9{\cdot}2$ per cent.

(c) Large items have less effect on it than in the arithmetic average.

Disadvantages.

(a) It is impossible to use it when any items are zero or negative.

(b) It is more difficult to calculate and less easily understood.

(c) It may locate a value which does not correspond with any actual example.

The Moving Average.—An application of the arithmetic average is the moving average.

The moving average is a simple method of indicating the trend measurement of data over a period.

To obtain the moving average a period is taken which is long enough to eliminate the effect of ordinary seasonal or recurring fluctuations.

Consider the data in Fig. 50.

Annual Profits of P.Q. Company

Year.	Profits.	5 years' Moving Average.
1	100	
2	200	–
3	400	600
4	700	2,582
5	1,600	4,442
6	10,010	5,562
7	9,500	6,340
8	6,000	7,020
9	4,590	6,098
10	5,000	5,218
11	5,400	5,078
12	5,100	5,280
13	5,300	5,400
14	5,600	—
15	5,900	—
	£65,400	

15 years average = £4360.

FIG. 50.—SERIES OF DATA SHOWING MOVING AVERAGE.

It is considered a five years' average will be representative. The first five years' profits are totalled and the average is taken. This figure is placed opposite the third year, *i.e.* the centre year of the five. In a 3- or 7-year average it would be placed opposite the 2nd and 4th year respectively. Next year's profit, £10,000, is included in the five years' total, and the £100 of the first year is dropped, and so on, *i.e.* one further year is included and the earliest of the previous five is dropped each succeeding year.

The result of these calculations is shown in the last column in Fig. 50, from which the trend of the company's earnings is discernible. Progress has been consistent after allowing for the abnormally profitable 6th, 7th, and 8th years. This can be shown in graphic form as will be illustrated in a later chapter dealing with trends.

The trend of turnover shown by the moving average enables the management to forecast the probable trend of sales for the near future, so that a reasonably reliable basis is obtained for framing future policy. A further advantage of this sales information is that a basis is obtained for budgeting and budgetary control in various directions.

The trend of expenses can be similarly reviewed, and if the ratio of expenses to sales and production should show a tendency of expenses to increase out of proportion, an inquiry would be prompted. Where a costing system is in operation, the causes will have been indicated already.

Probably not the least valuable use of the moving average is in regard to prices of raw materials and manufactured commodities generally, to indicate market conditions likely to affect buying of materials, selling prices and urnover. The trend of market conditions indicated by various moving averages is a valuable barometer which, unfortunately, is not used sufficiently by many business men. It is largely on such forecasts that some of the most successful businesses have been able to organise their policies.

The Progressive Average must not be confused with the moving average. When a progressive average is taken, the first figure in the series is not dropped, as is done for the moving average.

The Harmonic Average or Mean is the reciprocal of the arithmetic average of the reciprocals of the items. It is found by taking the arithmetic average of the fractions obtained by placing a numerator of 1 over each item, the average so calculated being the reciprocal of the harmonic mean. The use of these fractions can be avoided by using decimals, and tables of reciprocals in this form are available.

Example.—To find the harmonic mean of 8, 10, 4.

$$\left(\frac{1}{8} + \frac{1}{10} + \frac{1}{4}\right) \div 3$$

$$= \left(\frac{10}{80} + \frac{8}{80} + \frac{20}{80}\right) \div 3 = \frac{38}{80} \times \frac{1}{3} = \frac{38}{240}$$

then $$1 \div \frac{38}{240} = \frac{240}{38} = 6{\cdot}31, \text{ the harmonic mean.}$$

The harmonic mean is not of much use in business, but may be used for determining average performance, *e.g.* of alternating production, speeds or other data showing alternation of movement.

QUESTIONS

1. What is the purpose of averages? Name four kinds of average.

2. Write notes on: (1) weighted arithmetic average, (2) moving average.

3. Discuss the advantages and disadvantages of the arithmetic average.

4. What is the geometric average? For what purpose is it used?

5. Contrast the moving average and the progressive average. What is the purpose of the moving average?

6. What are the disadvantages of the arithmetic average as a type? (*London Association of Certified Accountants.*)

7. (*a*) Define geometric average and state concisely under five headings its characteristics. (*b*) In what direction, if any, is the geometric average particularly useful in the preparation of published statistics? (*London Association of Certified Accountants.*)

8. Find the average of the items 71·9, 83·7, 52·6, 97·3, 39·9, 72·0 when weighted with weights proportional to 2, 5, 1·5, 7, 4, 3.

What would be the approximate effect on the result if all the weights were increased by 0·5? (*Union of Lancashire and Cheshire Institutes.*)

9. Define a geometric average and state its characteristics. *Incorporated Accountants* (*Final*).)

10. Distinguish between the Arithmetic and Geometric mean. When should the latter be used? (*Incorporated Accountants* (*Final*).)

11. Why is the geometric mean used in the construction of index numbers of wholesale prices? (*Union of Lancashire and Cheshire Institutes.*)

12. State the advantages and disadvantages of the arithmetic average as a type. (*Incorporated Accountants* (*Final*).)

13. How is the "Harmonic mean" computed? Illustrate your answer by assuming that on a specific journey alternate miles are travelled by an aeroplane at 80 and 100 miles per hour. (*London Association of Certified Accountants.*)

14. The following is an extract from Table 14 of the Housing Volume of the 1931 Census, relating to the County Boroughs of Essex.

C.B.	Dwellings Occ. by Private Families.	Rooms Occ. by Private Families.	Private Families.	Population in Private Families.	Rooms Occ. Dwelling.	Families per Occ. Dwelling.	Persons per Family.	Persons per Room.
E. Ham	29,264		37,775	140,032	5·17		3·71	0·93
Southend		158,046		114,389	5·76	1·19	3·49	
W. Ham	49,280	253,254	72,904	289,042				

The figures for Essex A.C. including the C.B.'s are:—

	386,475	2,004,704	451,436	1,687,074	5·19	1·17	3·74	0·84

You are required to (*a*) complete the table relating to the C.B.'s by calculating and filling in the missing figures; (*b*) find the average number of persons per family for the three C.B.'s taken together. (*N.A.L.G.O.*)

CHAPTER XIV

AVERAGES: II. THE MEDIAN AND THE MODE

The Median.—The arithmetic average is not satisfactory for all purposes. For instance, it is difficult to measure certain qualities for which an average might be required, but for which no mathematical measure can be employed. It is usually impracticable to state definite measurement of such things as the ability of employees, advertising campaigns, endurance tests and other factors in business. Further, it is not always possible to obtain all the data necessary for the computation of the arithmetic or other calculated average. On the other hand, however, it may be possible, for instance, to arrange employees in order of competence or intelligence, or forms of advertising in order of results, and if such an array is made in order of size or importance from the lowest to the highest (or vice versa), the middle item may be regarded as the average one. In statistics this middle item of an array of data is called the median.

The median is a position average, and is the value of the mid-case in an array.

To Locate the Median all the items or numbers of a group or class are arranged in an array in order of size, the items are counted and the middle item is the required median. The median thus has the same number of items above it as below it.

The *position* of the median can be found by the formula $\frac{n+1}{2}$, in which n is the number of items in the array.

In the array arranged in Fig. 51 the median is the 7th item (viz. $\frac{n+1}{2} = \frac{13+1}{2} =$ 7th item), which means that the median average weekly sales in three months is £103.

If the number of items in the array is *even*, the practice is to take the mean of the two middle items, as the median must lie between them. One item removed from the array in Fig. 51 would make the median the 6½th term, *i.e.* between £102 and £103, with a value of $\frac{205}{2} =$ £102½.

Median in a Grouped Series of Items.—The procedure is similar to that just described. Two types will be considered: (*a*) When the items have been grouped by values or sizes (see Fig. 53) the median *value* or *size* is that of the group in which the median item falls. The numbers occurring are arranged in a cumulative total column, and the median is then $\frac{\text{Total} + 1}{2}$. For example, in the array in Fig. 53 the median size is the size of the $\frac{4609 + 1}{2}$ = 2305th item. By inspecting the cumulative totals, it is seen that this item falls in the total opposite size 8½, and this therefore is the median size, *i.e.* the median average size of shoes sold is 8½.

Week No.	Weekly Sales for 3 months.	Weekly Sales for 3 months.
	£	£
1	102	98
2	106	99
3	98	99
4	100	100
5	103	102
6	99	102
7	102	103 ← Median.
8	108	104
9	106	104
10	104	106
11	99	106
12	109	108
13	104	109

FIG. 51.—ORIGINAL DATA UNARRANGED.

FIG. 52.—SAME DATA IN A SIMPLE ARRAY IN ORDER OF MAGNITUDES.

The series in this example is a **Non-continuous or Discrete Series**—because it is set up in classes differing by definite amounts (sizes in half inches) known as class intervals.

(*b*) When the items have been grouped into classes, in which it may be assumed that the items in each class are equally distributed in order of size within each class, the median value can be arrived at as follows, using the data in Fig. 54. This data is based on an analysis of the pay-roll, and the median wage will be that of the $\frac{301 + 1}{2}$ = 151st item. From inspection of the cumulative total it will be seen that the 151st employee is in the third group (earning 20*s*. but under 25*s*. a week). As the employees at various wages in this group may be assumed to be equally distributed within the group, the median wage may be calculated in the following manner: count the number of items

X Co.

Numbers of Shoes Sold by Sizes in One Year

Size.	Number of Pairs.	Cumulative Total.
5	30	30
5½	40	70
6	50	120
6½	150	270
7	300	570
7½	600	1170
8	950	2120
M = 8½	820 Median in this total.	2940
9	750	3690
9½	440	4130
10	250	4380
10½	150	4530
11	40	4570
11½	39	4609
	Total 4609	

FIG. 53.

This array according to sizes is termed a Frequency Distribution which is Discrete or non-continuous.

X Co.

Number of Employees Engaged at Various Weekly Wage Rates

Wage Groups.	No. of Employees.	Cumulative Total.
10*s*. and under 15*s*.	30	30
15*s*. " 20*s*.	45	75
20*s*. " 25*s*.	150	225
25*s*. " 30*s*.	36	261
30*s*. " 35*s*.	24	285
35*s*. and over	16	301
	Total 301	

FIG. 54.—CONTINUOUS SERIES ARRANGED FOR FINDING MEDIAN WAGE.

(employees) below the median group, *i.e.* 30 + 45 = 75. Deduct this from the median term (151), *i.e.* 151 − 75 = 76; then in the group 20*s*. to 25*s*. in which there are 150 employees, the 76th is the median. Now, as there is a difference of 5*s*. between the lower and upper limit of the group, the median item may be regarded as being $\frac{76}{150}$ of 5 distance from the lower limit (20*s*.), *i.e.* the median wage is $20s. + \left(\frac{76}{150} \times 5\right) = 20s. + \frac{380}{150} = 22{\cdot}5s.$

This is to say, that the median average wage of the *X* Company's employees is £1 2*s*. 6*d*.

This procedure (known as **Interpolation**), if applied to a series of a discontinuous type (as in Fig. 53), may give only a fictitious value, as, for instance, a fractional size which cannot exist. Thus if sizes are in, say, half inches, it would be an obviously fictitious value if (in the previous example) a value were calculated for the 2305th item at something between 8 in. and 8½ in. In that series all the items in the eighth group must be size 8½ in. When instead of fixed sizes the items are continuous measurements like tons, gallons, earnings, etc., interpolation is often permissible.

Finding the Median Graphically. Plot the cumulative totals of the items on a graph, using the base line for the group intervals, and the vertical axis for the cumulative numbers. This is illustrated on page 95, based on the data in Fig. 54. Mark the median position on the vertical axis (151), and draw a dotted line from that point to intersect the curve: then a line dropped from the point of intersection to the base line indicates the value of the median on the base line scale, as in Fig. 57 (22·5*s*.).

Advantages and Disadvantages of the Median as an average or type:—

Advantages.

(*a*) If found directly, it is representative of an actual item.

(*b*) It is simple to understand.

(*c*) Extreme items do not affect its value.

(*d*) It can be obtained even when the values of all items are not known. Provided the middle items are known, and that there are the same number of larger and the same number of smaller items, the median can be located.

(*e*) It can be used for measuring qualities and factors to which mathematical measurement cannot be given.

Disadvantages.

(*a*) If the items are few, it is not likely to be representative.

(*b*) If the distribution is irregular, the location of the median may be indefinite.

(*c*) When the items are grouped (as into classes), it cannot be located with precision, particularly in a discrete series.

(*d*) The arranging of data in the necessary array is often tedious.

(*e*) It cannot be used to determine the total value of all the cases or items. The number of items multiplied by the median will not give the total of the data. It is not suitable for arithmetical calculations, and has but limited use in practical work.

The Mode.—The mode is what many people mean in ordinary conversation when they refer to such things as an average income, an average person, etc. An example which is most frequent or typical is implied. It is the term used to designate the most frequent item in a series; the value which occurs most in a group; or the position of greatest density. The mode may therefore be defined as the size of the variable that occurs most frequently, or the value about which most cases recur. The mode is the position of the maximum ordinate in a smoothed histogram. This is illustrated in Fig. 56.

Like the median, the mode has very limited practical use, and cannot be used for arithmetic manipulation. It is of value, however, for the purposes of, say, the maker of ready-made clothes, or the manufacturer of certain components or accessories in common use. In deciding upon wages and rates of pay, an employer will usually adopt the modal rate—the rate paid by most other employers. Many other practical points in business are dependent upon some modal feature, particularly in regard to wages, rents, prices, sizes of supplies and the like. Not infrequently modal information on many of these matters can be supplied straight away by persons of experience. On the other hand, the mode may not be well defined, and in some cases the "point of density" may be rather broad and the selection of the mode may be based merely on judgment or even desire.

In a frequency distribution it is the point of greatest frequency of occurrence.

Like the median, it is a position average, and for practical computation it is usually essential to arrange the data in a frequency distribution or array. If such a distribution arrangement does not show a well-defined mode or modal class the modal average should not be used.

How to Find the Mode (by Grouping).—The data are arrayed, and not infrequently the mode is at once apparent by inspection only, as in the data in Fig. 54, in which the group "20*s*. to 25*s*."

is obviously the largest, with 150 items in it. This, then, is the modal wage group.

When there is no apparent mode clearly revealed by the array, it is usually necessary to re-group the figures. This may be done by widening the classes or groups; a procedure which smooths out irregularities. This is shown in Fig. 55.

Numbers of Articles Sold by Sizes

Size.	Sales. (= Frequency.)					
1	2	9		22		
2	7		20		35	
3	13	28				48
4	15		35	60		
5	20	45			68	
6	25		**48**			**72**
7	23	**47**		**67**		
8	24		44		**67**	
9	20	43				58
10	23		38	52		
11	15	29			55	
12	14		40			59
13	26	45				
14	19					

FIG. 55.—LOCATION OF MODE BY GROUPING.

The procedure is as follows: the frequencies are grouped by twos; and again by twos, starting with the second item. Then they are grouped by threes three times, starting with the first item, then with the second, and lastly with the third. If necessary, grouping can be done in fours. As each set of groupings is completed, the maximum frequency is underlined or indicated in heavier figures. It will be seen that the black type figures include :—

The 6 group 3 times,
„ 7 group 5 times,
„ 8 group 3 times,

hence the mode is size 7, and the other most popular or most frequently sold sizes are sizes 6 and 8.

To Find the Mode (by Formula).—The following formula is useful to find the exact location of the mode in the modal group when the sizes of the items are in groups like those shown in Fig. 54, or, in fact, for any group series where there is a regular class interval.

The formula is :—

Let Z = mode,

l_1 and l_2 = the lower and upper limits of the modal group,

f_1 = the number of items or frequency in the modal group,

f_0 and f_2 = the frequencies in the next lower and next higher groups.

Then $Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2}(l_2 - l_1)$.

(*N.B.*—Sometimes c is used to represent $(l_2 - l_1)$.)

Substituting the values in Fig. 54 we have

$$Z = 20 + \frac{150 - 45}{300 - 45 - 36} \times (25 - 20) = 20 + \frac{105}{219} \times 5$$

= 22·4*s*., which is the modal wage.

Another formula often used is:

$$Z = l_1 + \frac{f_2}{f_2 + f_0}(l_2 - l_1)$$

also

$$Z = l_1 + c\left(\frac{f_2}{f_2 + f_0}\right)$$

where c represents $(l_2 - l_1)$.

Location of Mode by Use of a Smoothed Histogram.—This may be best shown by an example based on the following data, which give the number of orders taken of each of the sizes shown, and which may be plotted as a stepped curve as shown in Fig. 56.

Orders Taken.

Size.	Number.
2*s*. and under 4*s*.	9
4*s*. „ 6*s*.	27
6*s*. „ 8*s*.	45
8*s*. „ 10*s*.	54
10*s*. „ 12*s*.	42
12*s*. „ 14*s*.	30
14*s*. „ 16*s*.	9
	216

$$\text{Mode} = 8 + \frac{54 - 45}{108 - 87} \times (10 - 8) = 8{\cdot}8s.$$

In this graph the group intervals are plotted along the x-axis, and on each division is drawn a rectangle with its area proportionate to the number of items (orders) given in the table. The stair-case block diagram chart drawn in this way is called a Histogram. By drawing a free-hand smoothed curve as shown (approximately midway between points), a smoothed Histogram is formed.

The highest ordinate of this smoothed histogram indicates the mode, which, it will be seen, falls between 8*s*. and 10*s*., *i.e.* at 8·8*s*. This is verified by the calculation of the mode using the formula already given.

In drawing the smoothed line, as the smoothed histogram represents a continuous series it should begin on the base line, and normally (but not as an absolute rule) the curve should start about the middle of the next group before the minimum extreme and end about the middle of the space representing the last extreme measurement.

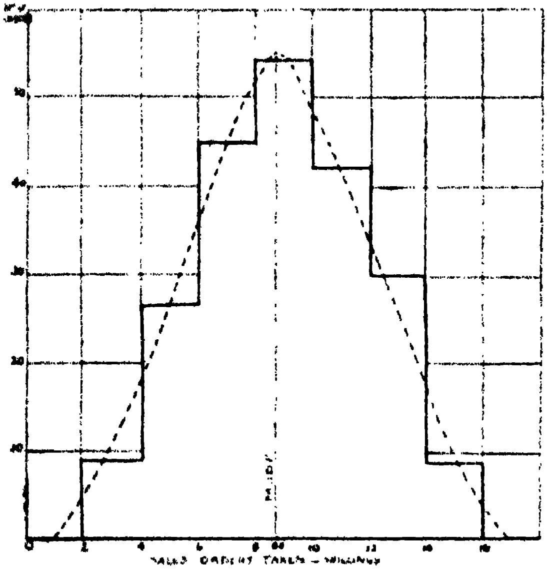


FIG. 56.—SMOOTHED HISTOGRAM SHOWING POSITION OF THE MODE. BASED ON DATA ON PAGE 93.

Series with More Than One Mode.—A series may have more than one mode, and is then said to be bi-modal, tri-modal, etc. In practice there is usually one prominent mode, with sub-modes, although several modes may be of equal importance. This might happen with data relating to the sale of goods, there being different modes for low-, medium-, and high-priced articles. By separating the data for each grade a uni-modal series for each of them is obtainable.

Advantages and Disadvantages of the Mode.

Advantages.

(*a*) It is easy to understand.

(*b*) Extreme items do not affect its value.

(*c*) Like the median only the middle items need be known.

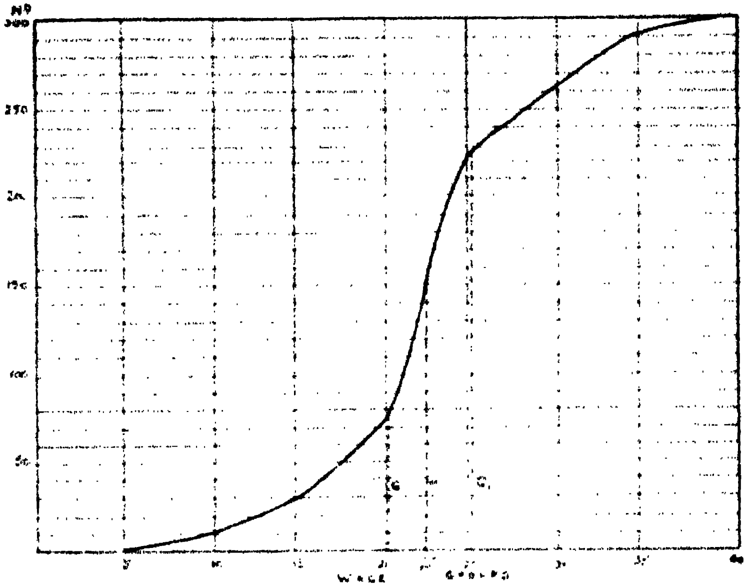


FIG. 57.—CUMULATIVE FREQUENCY CURVE (OR OGIVE) TO DETERMINE MEDIAN AND QUARTILES OF DATA IN FIG. 51.

Where the line crosses a point which is one-half the greatest ordinate, the reading on the x axis is the median. The quartiles are at one-quarter and three-quarters of the maximum ordinate.

Disadvantages.

(*a*) It is often not clearly defined.

(*b*) Exact location is often uncertain.

(*c*) Arrangement of data as an array, and the regrouping of items (when required) is tedious.

(*d*) It cannot be used for arithmetic calculations, as for instance the determination of the total number or value of all the items.

Quartiles, Deciles and Percentiles.—We have seen earlier that the median is the position average of a series which has been arranged in an array. The items above it in the array are larger, and those below it are smaller. If instead of taking the median items we divide the array into four divisions instead of two, the size of the items at three points in the array will be ascertained—namely, at the quarter, half and three-quarters. The sizes at these points are known respectively as the Quartile, Median and Third Quartile. Dividing the array in ten equal divisions, the sizes at each point are Deciles; and if in one hundred divisions, Percentiles, and similarly for other divisions of the array.

Thus if n is the number of items in the array :—

The Median is $\frac{n+1}{2}$

1st Quartile is $\frac{n+1}{4}$

3rd Quartile is $\frac{3(n+1)}{4}$

1st Quintile is $\frac{n+1}{5}$

1st Decile is $\frac{n+1}{10}$

8th Decile is $\frac{8(n+1)}{10}$

27th Percentile is $\frac{27(n+1)}{100}$ and so on.

In a group array the interpolation within a group can be calculated as for the median as explained on page 90.

Consider first the quartiles. They afford new information—namely, how much the items vary from the median, or, in other words, the quartiles express the spread or scattering of sizes or values from the median average. They are an aid to forming a rough idea of the amount of deviation from the median, which is often very useful information. The median gives a form of average size or value, and the quartiles give an idea of the tendency to deviation below and above that average. The first quartile is not necessarily of the same magnitude as the third—in fact, the deviation shown by the two quartiles is frequently different. It is only when the array reveals a symmetrical variation above and below the median that the quartiles are equal, and the median then lies exactly halfway between the lower and upper quartiles. When the variation is not symmetrical, we have what is known as Skewness, but this aspect of distribution will not be considered further at this stage except to point out that when the variation is greater towards the lower limit of a series, the Skewness is said to be negative; and towards the upper limit, positive. (See page 123.)

The position of quartiles, quintiles, deciles and percentiles in relation to the median can be found, and their values assessed, by means of a Frequency Curve drawn through points plotted for the *cumulative* values of the items (a cumulative frequency curve is called an Ogive). An example is given in Fig. 57.

It should be observed that the mode is found by plotting the actual values, not the cumulative ones.

In considering a frequency curve, it should be observed that the flatter the curve the closer are the quartiles to the size of the median.

An example of the use of quartiles in business is that of grading. Salesmen, factory workers, shop assistants, goods sold, etc., can be graded by making an array of the data concerning sales, production, hours, etc., but this use is very limited, and as a rule averages and index-numbers (described in the next chapter) are more informative.

QUESTIONS

1. From the data given, construct on *one* chart the Frequency Curves representing the distribution of the ages at marriage of males and females in England and Wales, 1931, and read off the modal ages.

Ages at Marriage.	Males.	Females.
	(Figures correct to nearest hundred.)	
Under 21 years . . .	136	494
21 but not 25 . .	979	1,206
25 ,, 30 . . .	1,183	850
30 ,, 35 . . .	378	257
35 ,, 45 . . .	222	179
45 ,, 55 . . .	108	76
55 years and upwards . .	97	41

(*N.A.L.G.O.*)

2.

Wt. in Kilos.	No. of Persons.	Wt. in Kilos.	No. of Persons.
65–69	1	40–44	7
60–64	2	35–39	11
55–59	4	30–34	5
50–54	7	25–29	4
45–49	15		

Find the median and the arithmetic mean of the above-given measurements. Plot a smooth curve for the frequency group. (*Building Societies Institute.*)

3. Plot a cumulative curve for the data in question 2 and indicate the position of the median and the two quartiles. (*Building Societies Institute.*)

4. What is the mode? What are its advantages and disadvantages?

5. Describe two ways of locating the mode.

6. Copy the smoothed histogram in Fig. 56 and insert lines to indicate the position of the median and arithmetic average.

7. Find the arithmetic mean and the approximate value of the mode of the following frequency distribution of the wages of 250 men.

Wages in Shillings.	Number of Men.	Wages in Shillings.	Number of Men.
Over 60	15	30-35	17
55-60	24	25-30	13
50-55	33	20-25	10
45-50	64	15-20	9
40-45	45	Under 15	1
35-40	19		

(*Union of Lancashire and Cheshire Institutes.*)

8. Draw a histogram and an ogive curve of the figures given in Question 7. Determine the values of the median and the quartiles graphically. (*Union of Lancashire and Cheshire Institutes.*)

9. State the advantages and disadvantages of the mode as a type. (*London Association of Certified Accountants.*)

10. Define (*a*) arithmetic average, (*b*) geometric average, (*c*) median, (*d*) mode, (*e*) quartile. Instance cases when (*b*), (*c*) and (*d*) are specially appropriate. (*Corporation of Accountants.*)

11. State the advantages and disadvantages of the mode. (*Incorporated Accountants* (*Final*).)

12. State the advantages and disadvantages of the median as a type. (*Incorporated Accountants* (*Final*).)

13. Determine (by grouping) the mode of the following series :—

m.	5	6	7	8	9	10	11
f.	36	40	45	52	54	49	48
m.	12	13	14	15	16	17	18
f.	47	53	52	47	43	42	45

(*Incorporated Accountants* (*Final*).)

14. Comment on the suitability of the class intervals below for use in connection with data, of which a sample item would be 29·9 :—

(A)	(B)	(C)
0–10	0–9·9	0–9·95
10–20	10–19·9	9·95–19·95
20–30	20–29·9	19·95–29·95
30–40	30–39·9	29·95–39·95
40–50	40–49·9	39·95–49·95

(*Incorporated Accountants (Final).*)

15. Why is a frequency table preferable to an average? (*Incorporated Accountants (Final).*)

16. Graph the following distribution of wages. Determine the arithmetic mean wage and the median; indicate their position in the graph.

Weekly Wages. s.	No. of Men.
30–40	11
40–50	26
50–60	63
60–70	81
70–80	35
80–90	21
90–100	13

(*London Chamber of Commerce.*)

17. State briefly and concisely the chief fallacies in the use of averages. (*London Association of Certified Accountants.*)

18. Locate by grouping the mode of the following frequency distribution :—

m.	*f.*	*m.*	*f.*
5	48	12	56
6	52	13	63
7	56	14	60
8	60	15	48
9	62	16	40
10	60	17	32
11	58		

(*Incorporated Accountants (Final).*)

19. Arrange the following data in a frequency distribution. Calculate the arithmetic average, median and mode.

Statistics of Weekly Wage-Earners

Weekly Wage.	No. of Wage-Earners.	Weekly Wage.	No. of Wage-Earners.
14·0	1	28·0	1
15·0	1	29·0	1
18·0	4	30·0	10
19·0	2	31·0	1
20·0	7	32·0	1
20·6	1	32·6	1
21·0	4	35·0	1
22·0	4	36·0	1
23·0	2	38·0	1
24·0	8	40·0	3
25·0	7	45·0	6
25·6	1	50·0	1
27·0	1	55·0	1
		Total	72

(*Incorporated Accountants* (*Final*).)

CHAPTER XV

INDEX NUMBERS

When it is desired to compare various forms of business activity and to ascertain what changes have taken place from one period to another as between various series of data, simple comparison and the use of averages are inadequate, and resort is made to the device of Index Numbers.

An Index Number is a statistical device for indicating the relative movements of data where measurement of the actual movements is difficult or incapable of being made.

In effect, data are made comparable by the introduction of a common denominator. To take a simple example, suppose that in 1930 the total of, say, the median (or average) price of 60 articles dealt in by a business is 300s., and a total taken in 1937 for 60 similar articles amounts to 390s. The ratio between the price total for 1937 and that for 1930 can be expressed as a percentage of the earlier year thus :—

$$\frac{390}{300} \times 100 = 130.$$

This is the index number, and shows prices to be 30 per cent. higher in 1937 as compared with the prices in 1930.

The Increasing Use of Index Numbers in Business management indicates that this method of analysis has practical value. Not only is this statistical device used for indicating relative price movements, in which field it is extensively employed, but also for data in relation to production, personnel and financial matters. These index numbers formerly used almost entirely in respect of general external conditions are now also frequently employed for measuring relative changes in internal data whereby the business executives have supplementary information to guide them.

The important thing to remember is that the index is not intended to show the course of movement of a single item during a period, but to give an indication of the general trend of some group or series of data as compared with some selected base. The great feature is that it gives an indication or estimation of the composite variation in many factors which affect changes in a series of data.

Preliminary Considerations in the Construction of an Index Number must be such that all factors are combined in a manner which will give each of them weight proportionate to its relative importance.

There are many methods of calculating index numbers, some of which will be described.

In the first place the following preliminaries of general application call for decision :--

1. The purpose must be considered.
2. What units or items, and how many of them, shall be taken into account ? For a price index, for instance, a suitable number of prices to be included must be decided upon. For general purposes 45 to 50 will serve, although for some purposes as many as 150 and 200 or even more are used.
3. Proper " weight," according to relative importance, of each of the items. Notice in Fig. 58 iron is included twice to provide a suitable weight.
4. The sources of the data to be included and the method of collecting them.
5. The base to be used as the reference or standard.
6. The form of average to be adopted.

Observations on some of these matters will show their import.

Choice of Items.--For convenience of description let us consider the selection of items for an index number of wholesale prices of raw materials.

All items cannot be included, but instead those most *representative* of each class in demand are selected, and particularly the ones which are most uniform as to type. The market prices for each material are obtainable from published reports and other sources.

With regard to the number of items used, it has been found that a larger number than about fifty gives only small gradual improvements in the index number for each increase in the number included. The Board of Trade Wholesale Price Index is now based on 200 items, whilst several American ones include numbers varying from 200 to 550. Specialised index numbers based on as few as 15 or 20 items are used by some business houses, and being calculated on definite data, are sensitive measures of relative price movements.

Methods of Weighting.—At this stage it may be noted that weights can be applied :—

(*a*) To the prices themselves, *e.g.* prices are multiplied by the physical quantities assigned to them and the products summated. (This is called the Weighted Aggregate Method.)

(*b*) To the price relatives, *i.e.* price relatives are weighted by expenditure or outlay. (This is called the Average of Ratios Method.)

In both cases the sum of the weights is used as the divisor. The illustration of these two methods given later will make the meaning clear, but it may be stated here that as errors in weights have but small effect there is no need for precision in weighting.

Sources of Data.—For general purposes published quotations (*e.g.* in trade journals) and those obtained from leading firms are often used. For index numbers relating to internal data of a business the cost, production and sales departments' records may be resorted to.

The Choice of Base may be :—

(a) *A Fixed Base.*—A particular year (or average of several years) is decided upon, and should be one of normal conditions. The base can be shifted without trouble when the relatives of aggregates method is used, but if the base is shifted when using the averages of ratios, the series used must be calculated afresh on the new base.

(b) *The Chain Base Method.*—Each year is calculated on the preceding year as a base, hence the results are chained together. Thus, using data in Fig. 58,

$$90{\cdot}5 \times \frac{84{\cdot}2}{100} = 76{\cdot}2$$

$$76{\cdot}2 \times \frac{148{\cdot}0}{100} = 112{\cdot}8, \text{ etc.}$$

This method is useful to business men in that (i) there is a direct comparison between one year and the next, which is more informative than a comparison with some distant earlier base year, and (ii) new items can be introduced or old ones eliminated.

The Type of Average to be Adopted.—Any form of average (arithmetic, geometric, median or mode) may be used, but for most purposes the arithmetic is the most practical, and, being more familiar, is better understood.

	Iron.	Iron.	Tin.	Lead.	Total.	Average.	Chain Index.
	(1)	(2)					
Year 1 Relative . .	80	82	119	81	362	90·5	90·5
„ 2 „ . .	60	64	128	62	314	78·5	76·2
(Year relative based on year 1) . . .	**75**	**78**	**108**	**76**	**337**	**84·2**	—
Year 3	102	105	111	106	424	106·0	112·8
(Year 3 relative based on year 2) . .	**170**	**164**	**87**	**171**	**592**	**148·0**	

FIG. 58.—THE CHAIN BASE INDEX NUMBER.

The geometric mean is difficult to calculate, and is much less intelligible, but it does not give undue weight to extremes, and is therefore often used for price indices (*e.g.* the Board of Trade Index of Wholesale Prices). The median and the mode are not advisable in business statistics, and particularly as these are very erratic.

THE CALCULATION OF INDEX NUMBERS

1. Unweighted Aggregative Method.—This method gives a simple aggregative index number, *i.e.* the ratio of the totals of prices.

The total of the values of the items is found for the base year and the current year. The latter is divided by the former and multiplied by 100, and the result is the index number.

Materials.	Units.	Base[1] year 1924.	Current[1] year 1930.
		£	£
Coal	Ton	1·6	1·2
Pig Iron . . .	Ton	4·5	3·5
Timber	1000 ft.	9·0	8·2
Copper	Ton	63·2	54·6
Tin	Ton	250·0	145·0
Lead	Ton	34·8	19·4
		363·1	231·9

[1] The values are assumed.

$$\text{Index number } \frac{231{\cdot}9}{363{\cdot}1} \times 100 = 63{\cdot}86.$$

FIG. 59.—CALCULATION OF UNWEIGHTED AGGREGATIVE INDEX NUMBER.

The example gives the index number of prices of six raw materials. It could be similarly compiled for, say,

The number of employees employed in a number of factories.

The sales of a selected number of businesses.

The output of so many factories or of workers in factory departments.

As compiled in this manner, unless the units included are of uniform importance, the more weighty items produce an undue influence. Thus a fairly large variation in the price of coal could not influence the index number with even a relatively small change in the price of lead. A 20 per cent. change in the latter would affect the total more than several hundred per cent. in the price of coal.

To overcome this, weights for, say, quantity may be employed

2. The Weighted Aggregative Method.—The procedure is as before, except that the items in both the base year and current year are multiplied by appropriate weights.

In the case of prices the weight used is ordinarily the quantity of each item produced or marketed. The quantities used as weights may be: (*a*) those of the base year, (*b*) those of the current year, (*c*) fixed weights estimated to be typical. Great accuracy in weighting is not necessary; errors in weights have less influence than errors in price or other form of evaluation.

Materials.	Base year. (1924.)	Current year. (1930.)	Weights.	Weighted Values, 1924.	Weighted Values, 1930.
	£	£	(In 000's.)	(£000's.)	(£000's.)
Coal . .	1·6	1·2	210	336	252
Pig Iron .	4·5	3·5	60	270	210
Timber .	9·0	8·2	30	270	246
Copper .	63·2	54·6	10	632	546
Tin . .	250·0	145·0	5	1,250	725
Lead . .	34·8	19·4	5	174	97
Total .				£2,932	£2,076

$$\text{Index number } \frac{2076}{2932} \times 100 = 70{\cdot}8.$$

FIG. 60.—WEIGHTED AGGREGATIVE INDEX NUMBER.

3. Average of Ratios or Percentage Relatives.—A relative is merely the ratio between the price (or other measure) of the current year (or other period) and that of the base period.

When the ratio is expressed as a percentage of the base period, it is called a percentage relative.

Thus: (*a*) Coal in 1924, price 32s.; in 1930, 24s. a ton, the percentage relative for coal in 1930 would be said to be $\frac{24}{32} \times 100 = 75·00$.

(*b*) Output in 1928, 1200 units; in 1936, 1600 units. Percentage relative or ratio for output in 1936 $= \frac{1600}{1200} \times 100 = 133·33$. The method of stating the base or reference of percentage ratios is as follows: 1924 = 100; 1928 = 100.

Applying the above to the data in Fig. 59, the percentage relative for the 6 items in 1924 would be 100, *i.e.* the price for each item is the base, and the percentage relatives or ratios for 1930 will be as shown in Fig. 61.

Materials.	Base Year.	Current Year.	Current Year, Percentage Relatives.
	£	£	
Coal . . .	1·6	1·2	75·0
Pig Iron . .	4·5	3·5	77·7
Timber . .	9·0	8·2	91·1
Copper . .	63·2	54·6	86·4
Tin . . .	250·0	145·0	58·0
Lead . . .	34·8	19·4	55·7
			6) 443·9
			73·98

FIG. 61 PERCENTAGE RELATIVES OR RATIOS, AND UNWEIGHTED ARITHMETIC AVERAGE.

If now the total of the current year price relatives is divided by the number of items, we obtain the **unweighted arithmetic average of ratios index number:**

$$\frac{443·9}{6} = 73·98.$$

To obtain the **unweighted median of ratios index number,** the percentage relatives are put into an array: 55·7, 58·0, 75·0, 77·7, 86·4, 90·9. The median is $\frac{75\cdot0 + 77\cdot7}{2} = 76\cdot3$. If the number of items were an uneven number, the median index number would be the middle item in the array.

To obtain the **unweighted geometric mean of ratios** index number, the logarithms of each of the relatives are totalled, and divided by the number of items. The antilogarithm of the quotient is then the required index number.

The Weighted Average of Ratios Index is found as follows :—

Multiply the percentage ratios by the weight assigned, total the products, and divide this total by the sum of the weights used.

Using the weights in Fig. 60 we have :—

Materials.	Base Year Prices.	Current Year Prices.	Weights.	Current Year Percentage Relative.	Product.
	£	£			
Coal . .	1·6	1·2	21	75·0	1575·0
Pig Iron .	4·5	3·5	6	77·7	466·2
Timber .	9·0	8·2	3	91·1	273·3
Copper .	63·2	54·6	1	86·4	86·4
Tin . .	250·0	145·0	0·5	58·0	29·0
Lead . .	34·8	19·4	0·5	55·7	27·8
Totals .			32		2457·7

FIG. 62. WEIGHTED AVERAGE OF RATIOS.

The weighted arithmetic average of ratios index number is therefore

$$\frac{2457\cdot7}{32} = 76\cdot80.$$

For the **Weighted Geometric Average of Ratios** the weights are applied to the logarithms, otherwise the calculation is the same. Although a very satisfactory index number is obtained by this method, it is not much used owing to the calculations necessary.

If **Monetary Values are used as Weights,** *i.e.* the total expenditure on each item in the group included for the purposes of the index number, the procedure is :—

Multiply the unit price of each item by the quantity and make a total ; this is the total expenditure, then express the total value of each item as a percentage of the total expenditure.

Trend of Data shown by Ratios.—In order to see the tendency of variations in a series of data over a period of years (months or other period) in relation to any commodity or other item, a series of percentage relatives or ratios may be prepared as shown in Fig. 63.

The quantity for each year (whether prices, quantities in tons, gallons, etc., or values, such as sales, production, etc.) is stated as a ratio of the selected base period, *i.e.*

$$\frac{\text{quantity for each year}}{\text{quantity for base year}} \times 100 = \text{percentage relative or ratio.}$$

Using the data in Fig. 63, taking 1928 as the base, the ratio for 1927 is $\frac{72{,}840}{33{,}096} \times 100 = 220$; for 1929 it is $\frac{53{,}760}{33{,}096} \times 100 = 163$. Should the base be changed, say to 1929, then the ratios would change, because each item would have the output for 1929 as divisor, 53,760; the result of such a change is shown in the column on the extreme right in Fig. 63. The series of production and the corresponding ratios may be shown effectively on a ratio-ruled graph.

Production of Castings by the X Company's Foundries, 1926–1936

Years.	Production (tons).	Percentage Relative or Ratio (1928 = 100).	Effect on Ratio by Changing Base to (1929 = 100).
1926	61,176	185	114
1927	72,840	220	135
1928	33,096	100	62
1929	53,760	163	100
1930	80,112	242	149
1931	62,208	188	116
1932	72,816	220	135
1933	78,144	236	145
1934	72,799	220	135
1935	76,122	230	141
1936	84,576	255	157

FIG. 63.—SERIES OF ACTUAL DATA AND RATIOS TO SHOW TREND OF PRODUCTION OUTPUT.

Trend of Data shown by Index Numbers.—The prices of sugar, tea, coffee and cocoa for six years are shown in Fig. 64 (imaginary prices). The procedure for compiling a table of index numbers to show the trend of prices of these articles as a whole is as follows :—

Select the base year, say, year 1 ; we could use an average of, say, three or more years' prices, but as the series is for only six years, the prices for year 1 will do.

Prices of Sugar, Tea, Coffee and Cocoa for Years 1 to 6

Year.	Sugar (per lb.).	Tea (per lb.).		Coffee (per lb.).		Cocoa (per lb.).	
	d.	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*
1	10	2	6	2	1	2	3
2	8	2	9	2	4	2	3
3	7	3	0	2	2	2	0
4	6	2	9	1	10	2	0
5	4½	2	6	1	10	1	8
6	4	2	0	1	6	1	3½

FIG. 64.—DATA FOR TABLE OF INDEX NUMBERS IN FIG. 65.

Now express the price of each article as a ratio of year 1. Thus for sugar, as the price in year 1 is 10*d.* = 100, the ratio for year 2 is $\frac{8}{10} \times 100 = 80$. Similarly for coffee, year 1, 2*s.* 1*d.* = 100, therefore year 5 ratio is $\frac{22d.}{25d.} \times 100 = 88$; and so on.

The ratios are now tabulated as in Fig. 65, and totalled (column 6), then this total is divided by 4 to obtain the average (column 7), which gives the desired index numbers for the six years.

Index Numbers for Sugar, Tea, Coffee and Cocoa for Years 1–6
(Year 1 = 100.)

Year.	Sugar.	Tea.	Coffee.	Cocoa.	Total.	Average, being Index Numbers.
1	100	100	100	100	400	100
2	80	110	112	100	402	100·5
3	70	120	104	89	383	95·7
4	60	110	88	89	347	86·7
5	45	100	88	74	307	76·7
6	40	80	72	61	253	63·2

FIG. 65.—UNWEIGHTED INDEX NUMBERS.

As all the articles are not of relatively equal value to consumers, weights must be applied. Assume that the relative importance is 4, 3, 1½ and 1½. It will be necessary to multiply each of the individual *ratios* by these weights, total (column 6 of Fig. 66) and divide the yearly totals by the total weights (10) instead of by the number of articles. This gives the **weighted average of ratios** (Fig. 66, column 7).

Weighted Index Numbers for Sugar, Tea, Coffee and Cocoa, Years 1–6
(Year 1 = 100.)

Year.	Sugar.	Tea.	Coffee.	Cocoa.	Total.	Weighted Index Nos.
1	400	300	150	150	1000	100·0
2	320	330	168	150	968	96·8
3	280	360	156	134	930	93·0
4	240	330	132	134	836	83·6
5	180	300	132	111	723	72·3
6	160	240	108	91	599	59·9

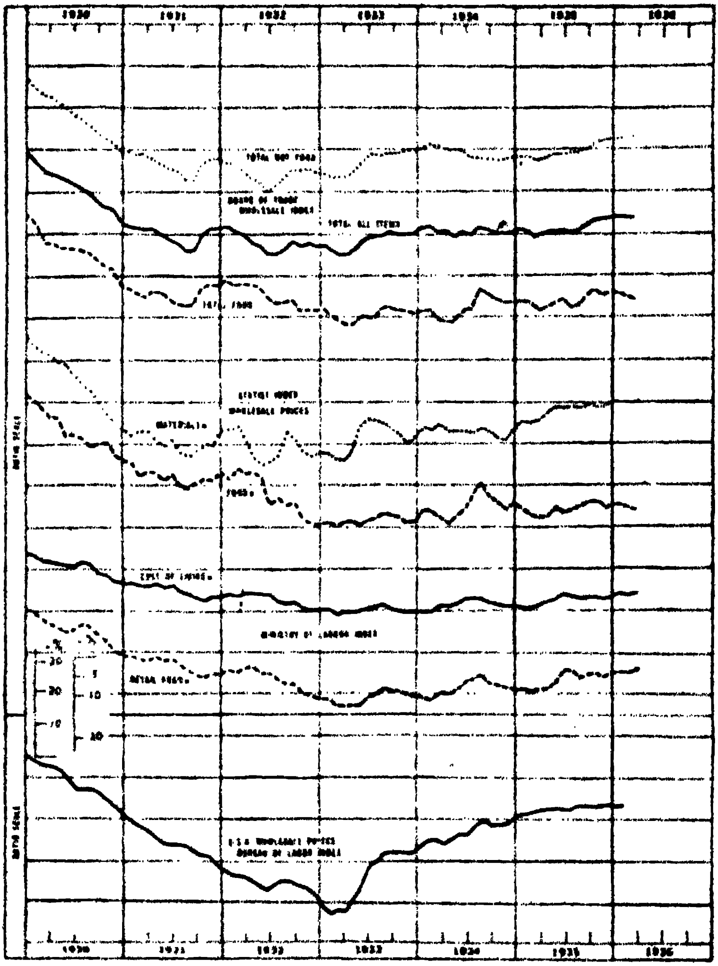
FIG. 66.—WEIGHTED INDEX OF DATA IN FIG. 65 USING WEIGHTS 4, 3, 1½, 1½, RESPECTIVELY.

Practical examples of published Index Numbers of Wholesale and Retail Prices in Great Britain and the United States are tabulated in Fig. 67*a*, and the corresponding curves are shown in the ratio-scale graphs in Fig. 67. For description, source, and comments on these, see page 161.

Index Numbers are Approximate Indications of conditions ruling only, excepting in the case of index numbers based on known internal data. All index numbers based on general data collected from selected reports, estimates and various sources can be only approximate, and further the following points must be taken into account when reading index numbers:—

1. The base may not be a normal period.
2. In a general index number, as all items cannot be included only the most important are used.
3. Prices and quantities may vary in different centres and an average may have to be used.
4. The weighting may have to be estimated, but, as previously stated, divergence in weights affects the index number less than errors in pricing. The weights are usually

PRICES.



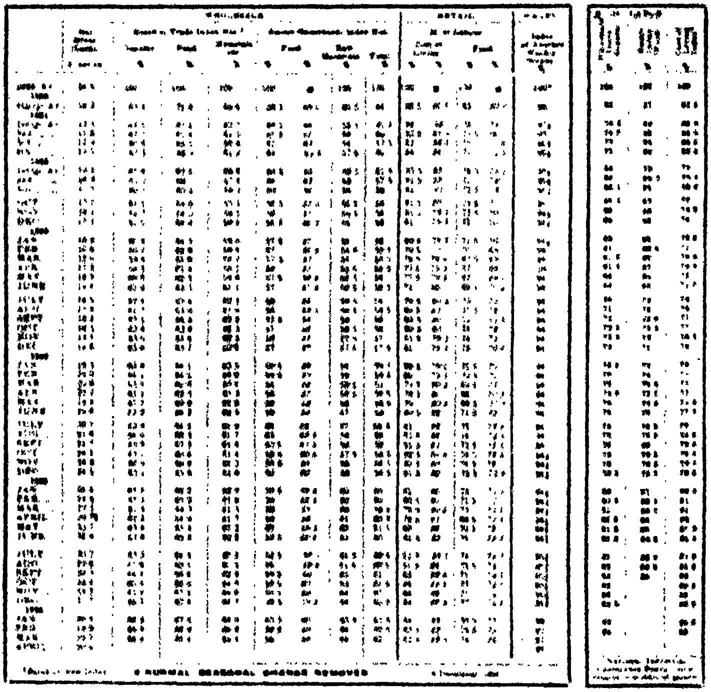
Scale applicable to all lines.

* NORMAL SEASONAL CHANGES REMOVED.

FIG. 67.—INDEX NUMBERS OF PRICES, PLOTTED FROM THE DATA IN FIG. 67a.

those of the base period, because as the base period has been selected for the conditions then ruling, the same contention can be made for the weights corresponding; but if other weights are more suitable, they should be used.

PRICES AND WAGES. U.S.A. PRICES.



PRICE OF SILVER	Average (cash) price of bar silver for week ending 15th of month.—*Economist.*
BOARD OF TRADE INDEX	Geometric Mean of Wholesale Prices (averages for month) of 200 commodities as percentage of 1924 average based on new index first published in January, 1935.—*Board of Trade Journal.*
"STATIST" (SAUERBECK) INDICES	Average wholesale prices of 15 foodstuffs and 20 raw materials on last day of month, as percentage of average for 1924.—*Statist.*
COST OF LIVING INDEX	Ministry of Labour's index showing movement since 1914 in cost of maintaining unchanged the standard of living prevalent in working-class households before the war. For 1st of month, but placed against previous month—e.g. reading for March 1st is shown against *February*—to facilitate comparison with *Statist* index.
RETAIL FOOD PRICES	As above, for food only.

FIG. 67a.—TABLE OF INDEX NUMBERS OF WHOLESALE AND RETAIL PRICES AND WAGES.

In conclusion, whichever method is used, the trend of movement will be in the same direction, and all are safe indications without being precise. When a larger number of items is included in the calculations, the index number is more reliable, and except those for specific internal purposes, index numbers should be based on not less than about 40 to 50 items.

It is unwise to use an index number for purposes other than that for which it was designed, as almost without exception some erroneous interpretation may arise. The application of an index number to inquiries, even those cognate to the purpose, requires careful consideration.

QUESTIONS

1. Explain and illustrate the meaning of Index Numbers. Show the convenience which may result from their use. What special problems are presented by the determination and employment of index numbers of the Cost of Living? (*London Chamber of Commerce.*)

2. Give an account of the official method of measuring changes in the retail prices of working class "necessaries of life." (*London Chamber of Commerce.*)

3. What matters must be considered and decided upon preliminary to constructing an index number of (say) wholesale prices?

4. Write notes on the choice of items for an index number and the methods of weighting them.

5. Explain the meaning of "weighted average of ratios."

6. What factors should be considered when reading index numbers before using them for a particular purpose?

7. It has been stated that in the computation of index numbers generally, many problems present themselves. State concisely six points which you consider would arise in this connection. (*London Association of Certified Accountants.*)

8. From the following figures, which show the distribution of income percentage expended by an ordinary middle class family, calculate the index of the cost of living for the year 1929, taking 1928 as a base year of 100:—

Food	35	per cent.
Rent	15	,,
Clothing	20	,,
Fuel and light	10	,,
Miscellaneous	20	,,
Total	100	,,

The average prices during the two years were as follows:—

Year.	Food.	Rent.	Clothing.	Fuel and Light.	Miscellaneous.
1928	£150	£30	£75	£25	£40
1929	145	30	65	23	45

(*London Association of Certified Accountants.*)

9. What method would you use, and what statistics would you require, to test the movement of prosperity in any one industry over a period of (say) 10 years?

(Incorporated Accountants (Final).)

10. Sketch the lines on which you would proceed to prepare an index number of the cost of living.

(Incorporated Accountants (Final).)

11. You are given the following series of Index Numbers of prices of four commodities and a straight index of the four taken together, based on the average. Calculate a new Index-Number for the six years based on the chain method.

Year.	Sugar.	Milk.	Coffee.	Tea.	Total.	Average.
1921	81	77	119	55	332	83·0
1922	62	54	128	82	326	81·5
1923	104	87	111	100	402	100·5
1924	93	75	154	96	418	104·5
1925	60	43	165	88	356	89·0
1926	60	44	159	89	352	88·0
1927	62	47	139	84	332	83·0

(Incorporated Accountants (Final).)

12. From the following data construct Index Numbers showing the change in the average price of Corn Crops, 1921–1931, (*a*) using the quantities of 1921, (*b*) using the quantities of 1931. Comment briefly on the main problems involved in the construction and use of the type of Index Number you have calculated.

Crop.	Estimated Total Produce, U.K. (in 000 tons).		Average Annual Price per cwt.	
	1921	1931.	1921.	1931
			s. *d.*	*s.* *d.*
Oats . .	2,142	1,861	12 3	6 3
Wheat . .	2,027	1,010	16 8	5 9
Barley . .	1,171	847	14 7	7 11

(N.A.L.G.O.)

13. Using the following data compute an index of real wages for the years indicated:

	Index of Money Wages.	Index of Cost of Living.
1913 . . .	100	100
1914 . . .	102	102
1915 . . .	104	104
1916 . . .	118	111
1917 . . .	134	133
1918 . . .	168	159
1919 . . .	193	183
1920 . . .	232	208
1921 . . .	207	182
1922 . . .	201	168
1923 . . .	220	171

(*Building Societies Institute.*)

14. United Kingdom: Imports of dutiable beverages, 1906 and 1919

		1906.		1919.	
		Quantities.	Values (£).	Quantities.	Values (£).
Beer and Ale	barrels .	58,577	158,650	418	4,057
Cocoa, raw .	lbs. .	51,670,321	1,335,107	246,623,216	8,943,025
Coffee .	cwts. .	765,561	2,021,648	1,066,046	5,988,812
Tea .	lbs. .	321,190,064	9,961,085	404,353,466	33,050,853
Wine	gallons	13,103,364	4,214,878	25,252,387	18,167,077

Using the statistics given above, calculate two index-numbers of the change from 1906 to 1919 in the physical volume of dutiable beverages imported, using (i) weights appropriate to the relative importance of the different beverages in 1906, and (ii) weights appropriate to 1919.
(*N.A.L.G.O.*)

CHAPTER XVI

MEASURING VARIATIONS FROM THE AVERAGE

How Well does an Average Represent the Group.—When an average of any kind is considered to represent a particular group of items, there will be many of the items which do not coincide with that average. For most business purposes the arithmetic average is sufficiently useful, although for certain types of business estimating, the median and the mode have advantages. The *extent* to which items in a group or series vary from the average is information which is not often required, but for some purposes there is practical value in ascertaining the range of variation and some kind of average of variation from the selected average of the items. In statistical terminology the variations are referred to as Dispersion.

Dispersion is the variation of the separate items in a group from their average. Its purpose is to show the representativeness of the average. If there is not much dispersion, then the average is a good representation of the group.

Four Methods of Measuring Dispersion will be briefly described, although there are many others which need not be considered.

The four methods are :

1. Quartile deviation, which has been explained already in Chapter XIV.
2. The Range, *i.e.* the limits of the highest and lowest items in a group.
3. The Average Deviation.
4. Standard Deviation.

The Range, or limits within which lie all the items in a group, gives but broad information. Thus in Fig. 52 the range shows : (i) That the lowest amount of weekly sales in thirteen weeks was £98 and the largest £109. (ii) That the maximum variation was £11 (*i.e.* £109 to £98). (iii) That the variation of the lowest item from the arithmetic average for the period (£103) was £5, and of the largest item was £6.

This measure of dispersion loses value in that other variations within the extreme limits are not indicated, yet the items may vary in many ways within those limits. It has practical value, however, where a rough idea is required of items within prescribed limits. For instance, in analysing output of workers it is found (see Fig. 68) that in the N. Factory they range from (say) 100 to 140 units per day, the average being 119, and most results range from 110 to 120 units per day. If the output from the S. factory is reviewed in the same way, the results may be compared with those of the first named, viz. range 90 to 130 units per day, average output 112, and most results range from 105 to 115 units per day.

The range and the quartiles therefore give useful rough measurements of the possible extreme limits of variation, but these are too indefinite to be of much practical value.

Output of Piece-workers

N. Factory.		S. Factory.	
Output in Units.	No. of Workers Producing.	Output in Units.	No. of Workers Producing.
100	8	90	9
105	16	95	10
110	34	100	16
115	70	105	40
120	100	110	56
125	30	115	110
130	20	120	27
135	12	125	20
140	10	130	12
	300		300

FIG. 68.—RANGE OF OUTPUT IN TWO FACTORIES.

The Quartile Deviation, unlike the range, is not affected by the size of the extreme items, as it is the measure of the spread of the items between the quartiles. It is the measure of the items over a restricted range, and to that extent resembles the range deviation. In fact, the quartile deviation is sometimes called the **Semi-interquartile Range,** for the reason that its *value* is half the difference between the third quartile and the first, viz. :—

$$\text{Absolute Quartile Deviation} = \frac{\text{Value of 3rd Q} - \text{Value of 1st Q}}{2}$$

Applying the data in Fig. 69,

$$\text{absolute Quartile Deviation} = \frac{71 - 67}{2} = 2d.$$

Bonus Payments to Pieceworkers

Bonus in Pence.	No. of Workers.	Cumulative.
65	20	20
66	17	37
67	31	68
68	40	108
69	25	133
70	53	186
71	25	211
72	20	231
73	20	251
	251	

Median $\frac{251 + 1}{2}$ = 126th worker. Group 69*d.*

Quartile 1 = 63rd worker. Group 67*d.*
Quartile 3 = 189th worker. Group 71*d.*

FIG. 69 TABLE ARRANGED TO FIND QUARTILES.

The Relative Quartile Deviation is found by dividing the quartile deviation by the sum or average of the two quartiles, viz. :—

$$\frac{\frac{Q_3 - Q_1}{2}}{\frac{Q_3 + Q_1}{2}} = \frac{Q_3 - Q_1}{Q_3 + Q_1}.$$

This relative quartile deviation is called the **Quartile Coefficient of Dispersion.**

Using again the figures in Fig. 69,

$$\text{the Coefficient of Dispersion} = \frac{71 - 67}{71 + 67} = 0{\cdot}029$$

(*i.e.* Relative Quartile Deviation).

Average Deviation.—When it is desired to find a closer measurement of the deviation of all items from the average of the items (arithmetic, median or mode), the simplest is the Average Deviation. The symbol usually used is *d*. This is the *First Moment of Dispersion.*

The average deviation is simply an arithmetic average of the deviations from an average of the items. The difference between the items in a series and an average of the items is ascertained, and then the average of differences. (N.B.—Signs are to be disregarded.) It is usual to use the arithmetic average.

The following formulæ represent the First Moment of Dispersion:

(*a*) from average $\frac{\Sigma d}{n}$; (*b*) from median $\frac{\Sigma d_M}{n}$; (*c*) from mode $\frac{\Sigma d_Z}{n}$.

The mode is seldom used.

In the case of a grouped series, we take the difference between the average and the middle point of each individual group or class.

Using the data in Fig. 69, the arithmetic average of the bonus payments is 69*d*., and the differences between this and each of the items in the series are as shown in Fig. 70, column 2.

Bonus. (1)	Deviations from Average (69). (2)	No. of Workers (Weights). (3)	Products. (4)	Squared Deviations. (5)	Products (Sqd. Deviations × Weights). (6)
d.					
65	4	20	80	16	320
66	3	17	51	9	153
67	2	31	62	4	124
68	1	40	40	1	40
69	0	25	0	0	0
70	1	53	53	1	53
71	2	25	50	4	100
72	3	20	60	9	180
73	4	20	80	16	320
9)621		251	476		1290
Average 69		(= n)	(= Σd)		

FIG. 70.—COMPUTATIONS FOR FINDING AVERAGE DEVIATION AND COEFFICIENT OF DISPERSION (COLUMNS 1 TO 4). ALSO STANDARD DEVIATION (COLUMNS 3, 5 AND 6).

These deviations are multiplied by the weights, in this case the number of workers of each group, and the average of the products, $\left(\frac{476}{251} = 1{\cdot}896\right)$, is the Average Deviation. That is, the average deviation from the average bonus of 69*d*. is 1·896*d*.

The conversion of this absolute average (usually written δ) which is a concrete measure to a relative one is as follows :—

$$\frac{1{\cdot}896}{69} = 0{\cdot}0275 \text{ or } 2{\cdot}75\% \ (= \text{Coefficient of Dispersion}),$$

which signifies that the items vary individually from the average (69*d.*) to the average extent of 2·75 per cent. By ascertaining the average coefficient of dispersion in this way, two or more groups of figures, say bonus payments earned in each of two factories, the respective variations are easily comparable. When the magnitudes of the actual figures are great, or units are different, comparison by means of relative dispersion is very convenient.

The formulæ for the above coefficients of dispersion are :

$$\frac{\delta}{a}\,;\quad \frac{\delta_M}{M}\,;\quad \frac{\delta_Z}{Z}$$

and it should be observed that these coefficients are abstract measures; δ, δ_M and δ_Z are of course concrete.

Standard Deviation.—The method of calculating the standard deviation only differs from that for average deviation in that the deviations from the average are squared before they are multiplied and totalled. This is the *Second Moment of Dispersion*.

It is the best measure of dispersion as it is more reliable. A great advantage is that it can be used for further arithmetic operations, whereas the mean or average deviation cannot.

Hence the procedure is :—

(*a*) Find the arithmetic average of the series of items (*i.e.* 69*d.* using data in Fig. 69).

(*b*) Find the deviation of each item in the series from that average (Fig. 70, column 2).

(*c*) Square each of these deviations (Fig. 70, column 5).

(*d*) Total these squares and divide by the number of frequencies in the group (*e.g.* employees) *or* if the series is a grouped one, multiply the squares by the frequencies, total and divide by the frequencies.

(*e*) Take the square root of the result of (*d*) and thus find the Standard Deviation, or as it is sometimes called, the Root-Mean Square Deviation.

Standard Deviation =

$$\sqrt{\frac{\text{Sum of squared deviations from the arithmetic average,}}{\text{Number of items in entire group}}}$$

or :—

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}}$$

The coefficient is calculated in the same manner as before, viz :—

$$\frac{\sigma}{a}$$

Applying the data in Fig. 70

$$\text{Standard Deviation} = \sqrt{\frac{1290}{251}} = \sqrt{5{\cdot}139}$$

$$= 2{\cdot}27$$

$$\text{Relative dispersion} = \frac{2{\cdot}27}{69} = 0{\cdot}033$$

or 3·3 per cent. (= Coefficient of Dispersion, or Coefficient of Variation).

A Short Method of preparing data for finding the standard deviation is to take an approximate assumed average. By doing this a whole number may be used, thus avoiding the troublesome calculations involved when the real average is not a round number. The procedure is shown by the following example :—

Wages.	Deviation from Assumed Average (30s.).	No. of Employees (Frequency) n.	Product (Col. (b) × Col. (c)).	Product (Col. (b) × Col. (d)).
(a)	(b)	(c)	(d)	(e)
s.	Units.			
20	− 2	2	− 4	8
25	− 1	22	− 22	22
30	0	20	0	0
35	+ 1	10	+ 10	10
40	+ 2	5	+ 10	20
45	+ 3	6	+ 18	54
50	+ 4	3	+ 12	48
55	+ 5	1	+ 5	25
60	+ 6	1	+ 6	36
		70	+ 35	223

Fig. 71.—Wages Arranged in Units of Five Shillings with Computations for Finding Standard Deviation.

Say assumed arithmetic average is 30s., *i.e.* 6 units of 5s.

Then the actual arithmetic average may be simply arrived at thus :—

$$6 + \frac{35}{70} = 6\tfrac{1}{2}\text{ units} = 32s.\ 6d.$$

and the standard deviation thus :—

$$\sqrt{\frac{223}{70} - \left(\frac{35}{70}\right)^2} = \sqrt{\frac{223 \times 70 - 35 \times 35}{70 \times 70}}$$

$$= \sqrt{\frac{15{,}610 - 1{,}225}{4{,}900}} \text{ units of } 5s.$$

$$\sqrt{2{\cdot}93} = 1{\cdot}714 \text{ units}$$

$1{\cdot}714 \times 5s. = 8{\cdot}57s.$ Standard Deviation.

The formula for this may be written

$$\sigma = \sqrt{\frac{\Sigma d_x^2 - n(a - x)^2}{n}}$$

Where x = assumed average, a = actual average.

The Third Moment of Dispersion is found from the formula $\frac{\Sigma d^3}{n}$; and another measure, the *Modulus* (usually written c) from the formula $c = \sqrt{\frac{2\Sigma d^2}{n}}$.

Use in Business of Average and Standard Deviation Measurements.—These are valuable for comparing series of data in which there are wide variations over a period of time, and also for compilation of a business index composed of a number of series of business data. Not infrequently a small variation in a series of data relating to one commodity, and wide variations in a series of another related commodity, occur during the same period. If the two were averaged, the pronounced variations of the one commodity would have an overwhelming influence on the trend indicated for these two commodities. If, however, the data are represented by the standard deviations in each case, and averaged, the resulting trend indicated will be influenced equally by the fluctuations of both commodities. For the purpose of an index of business conditions this is what is required, as movements of sales, prices, etc., of all commodities comprised in the index are equally significant.

An example is afforded in Fig. 72, the calculations in which are apparent. The wider variations in artificial silk are reflected by the figures in columns 1 and 5, whilst the figures in columns 4 and 8 show how many standard deviations under or over the average sales occur month by month.

Graphs showing Dispersion.—In a frequency graph, greater dispersion is shown by the spread of the curve, *i.e.* when the items showing deviation are well scattered through the series the curve shows a succession of peaks of various heights, but when the items are concentrated about a certain level the curve exhibits a pronounced peak at that point.

	Sales of Silk Articles.				Sales of Artificial Silk Articles.			
	Sales (000's). (1)	Deviations from Average. (2)	Deviations Squared. (3)	Standard Deviations from Average Sales (4)	Sales (000's) (5)	Deviations from Average (6)	Deviations Squared (7)	Standard Deviations from Average Sales. (8)
	£				£			
January	2	0	0	0	2	−6	36	−1·1
February	1	−1	1	−1·4	2	−6	36	−1·1
March	1	−1	1	−1·4	10	+2	4	+0·4
April	2	0	0	0	15	+7	49	+1·3
May	3	+1	1	+1·4	18	+10	100	+1·8
June	3	+1	1	+1·4	4	−4	16	−0·7
July	3	+1	1	+1·4	4	−4	16	−0·7
August	2	0	0	0	4	−4	16	−0·7
September	2	0	0	0	15	+7	49	+1·3
October	2	0	0	0	10	+2	4	+0·4
November	2	0	0	0	10	+2	4	+0·4
December	1	−1	1	−1·4	2	−6	36	−1·1
Total	24		6		96		366	
Monthly Average	2		0·5		8		30·5	
Standard Deviation	√0·5 = 0·7, i.e. £700.				√30·5 = 5·5, i.e. £5500.			

Note.—The figures in columns (4) and (8) are obtained by dividing each item in column (2) and (6) respectively by the standard deviations 0·7 and 5·5.

FIG. 72.—COMPARISON OF TWO TIME SERIES OF SALES IN TERMS OF THEIR STANDARD DEVIATIONS.

The Lorenz Curve is a graphic method of measuring divergence from the average, and although devised by Lorenz for measuring distribution of wealth, it may be applied to compare the distribution of profits over different groups of businesses, or the distribution of sales over classified customers in two areas, etc.

It is a cumulative percentage curve which combines the percentage of the items with the percentage of (say) wealth, profit, sales and other factors distributed among those items.

Take as an example the turnover of businesses in the "X" trade, hypothetical data of which are shown in Fig. 73, and the corresponding Lorenz Curve in Fig. 74. The curve shows the concentration of turnovers diverging from the average, for if all the businesses had the same turnover, the straight line *xy* would represent that equal distribution. The more concave the curve the greater the concentration of items divergent from the average. The curve is not as valuable as the coefficient of dispersion, as it does not furnish any numerical result.

Turnovers and Numbers of Businesses in the X Trade, 19...

Turnover Exceeding (000's). (a)	No. of Businesses. (b)	Cumulative Total of Col. (a). (c)	Col. (c) as Percentage of Total. (d)	Cumulative Total of Col. (b). (e)	Col. (e) as Percentage of Total. (f)
£		£			
1	26	1	1·0	26	13
2	32	3	3·0	58	29
4	42	7	7·0	100	50
8	46	15	15·0	146	73
10	18	25	25·0	164	82
20	16	45	45·0	180	90
25	14	70	70·0	194	97
30	6	100	100·0	200	100

FIG. 73.—DATA PLOTTED AS LORENZ CURVE IN FIG. 74. COLUMN (d) IS PLOTTED ON VERTICAL SCALE; COLUMN (f) ON HORIZONTAL.

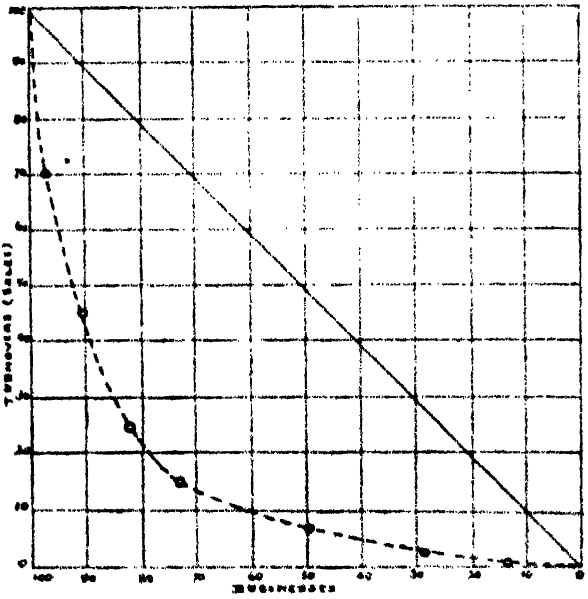


FIG. 74.—EXAMPLE OF LORENZ CURVE SHOWING DIVERGENCES OF TURNOVER FROM AVERAGE OF 200 BUSINESSES. NOTE THE REVERSAL OF THE SCALE ON THE BASE LINE.

Skewness is lack of symmetry in a frequency distribution. When the data are plotted on a chart and skewness is present, the items tend to be dispersed more on one side of the mean than on the other. In an absolutely symmetrical distribution the median, mode and arithmetic average coincide, and skewness is 0, but when the curve is skewed, there is a normal sequence of mode, median and arithmetic average, the two latter moving to the left or right, according to which direction the curve is skewed. The mode remains in the same position.

In a graph the sequence of the three forms of average (Mode, Median and Arithmetic Mean) always remains in the same order, viz., mode median, arithmetic mean, however much the degree of skewness may vary.

Hence, in negative skewness, the order starting from the mode will be: mode, median, mean, and in positive skewness it will also be mode, median, mean.

As a general rule, when the curve is not highly skewed, the median will move only about two thirds the distance of the arithmetic mean.

Skewness to the right on the curve is positive; to the left is negative.

The measure or coefficient of skewness (symbol usually used, "j") may be obtained from one or other of the following formulæ :—

(*a*) 1st Coefficient of skewness $= \dfrac{\text{Arithmetic mean} - \text{Mode}}{\text{Standard (or mean) deviation}}$

$$i.e.\ j = \frac{a - Z}{\delta} \text{ or } j = \frac{a - Z}{\sigma}.$$

If the mode is badly defined we may substitute—

$$\text{Coefficient of skewness} = \frac{3(\text{Arithmetic mean} - \text{Median})}{\text{Standard deviation}},$$

$$i.e.\quad \frac{3(a - M)}{\sigma}.$$

(*b*) 2nd Coefficient of skewness =

$$\frac{\text{3rd Quartile} + \text{1st Quartile} - 2\text{ Median}}{\tfrac{1}{2}(\text{3rd Quartile} - \text{1st Quartile})},\ i.e.\ \frac{Q_3 + Q_1 - 2M}{\tfrac{1}{2}(Q_3 - Q_1)}.$$

This formula ignores the size of extreme items.

(c) The following measure allows for the extremes, viz.,

$\sqrt[3]{\frac{\Sigma d^3}{n}}$ and using (i) the average deviation and (ii) standard deviation the third coefficient would be :—

$$\text{(i) } j = \sqrt[3]{\frac{\Sigma d^3}{n}} \div \delta \qquad \text{(ii) } j = \sqrt[3]{\frac{\Sigma d^3}{n}} \div \sigma$$

(*a*) and (*b*) are unreliable if the mode or median is badly defined, as is often the case ; (*b*) is simple but is unreliable when the median and quartiles are not clearly defined. It also fails to allow for extreme variations.

The object of determining skewness is to find (*a*) whether it is positive or negative, and (*b*) what percentage of the items are above or below the average. Two series may have similar averages and the same amount of dispersion, and nevertheless differ considerably according to the direction of skewness. Otherwise there is not much value in the measure of skewness in practical work.

QUESTIONS

1. What do you understand by "dispersion" and what are the different ways of measuring it ? (*London Chamber of Commerce.*)

2. Give the formulæ for computing Standard Deviation by the short-cut method. (*London Association of Certified Accountants.*)

3. Compute the quartile coefficient of dispersion of the following array of identities :

Frequency	3,	10,	12,	15,	17,	22,	21,	20,	18,	12,	6,	4,	3.
Size of item	5,	6,	7,	8,	9,	10,	11,	12,	13,	14,	15,	16,	17.

(*London Association of Certified Accountants.*)

4. Show the average deviation in the following series relating to factory output, and calculate the coefficient of dispersion :—

Daily Output. No. of Articles.	No. of Workers.	Daily Output. No. of Articles.	No. of Workers.
100	4	116	12
102	2	118	72
104	6	120	16
106	4	122	40
108	20	124	36
110	10	126	18
112	30	128	20
114	40		

5. Calculate the standard deviation from the data given in question 3, and show the coefficient of dispersion.

6. Calculate to 2 decimal places the standard deviation of the following distribution :—

m.		*f.*
Exceeding.	Not Exceeding.	
4·5	5·5	2
5·5	6·5	9
6·5	7·5	11
7·5	8·5	12
8·5	9·5	10
9·5	10·5	13
10·5	11·5	6
11·5	12·5	4
12·5	13·5	1

(*Incorporated Accountants* (*Final*).)

7. Define: (*a*) Mode. (*b*) Median. (*c*) Average deviation. (*d*) Standard deviation. (*e*) Percentile. (*f*) Historigram. (*Incorporated Accountants* (*Final*).)

8. Calculate the quartile coefficient of dispersion of the following array:—

Mid Point of Class Measurement.	*f.*	Mid Point of Class Measurement.	*f.*
1	4	9	16
2	6	10	14
3	9	11	10
4	11	12	7
5	15	13	5
6	20	14	3
7	25	15	1
8	19		

State the considerations on which the construction of this measure of dispersion is based. (*Incorporated Accountants* (*Final*).)

9. What do you understand by standard deviation? What practical use can be made of it?

10. Calculate the standard deviation of the following data:—

Exceeding. *m.*	Not Exceeding. *m.*	*f.*
5½	6½	4
6½	7½	2
7½	8½	5
8½	9½	7
9½	10½	9
10½	11½	4
11½	12½	2

(*Incorporated Accountants* (*Final*).)

11. What is the effect of skewness on the sequence of averages? Illustrate your answer by means of a histogram. (*London Association of Certified Accountants.*)

12. What is meant by the term "skewness"? Illustrate your answer by means of a histogram. (*London Association of Certified Accountants.*)

13. What are the methods used for measuring dispersion, and which method discloses the most accurate result? Apply your answer to the profits of two companies for three years, viz.:—

	Net Profit. 1st Year.	Net Profit. 2nd Year.	Net Profit. 3rd Year.
A Company . . .	£23,500	£25,000	£26,500
B Company . . .	48,500	50,000	51,500

(*London Association of Certified Accountants.*)

14. What are the characteristics of the coefficient of dispersion, and in what respects may it be used with advantage? (*London Association of Certified Accountants.*)

15. Define skewness. State the formulæ for the various coefficients of skewness, adding notes on their respective advantages and disadvantages. Use the following symbols: a = arithmetic average, M = median, Z = mode, Q = quartile, d = deviation. (*Incorporated Accountants (Final).*)

16. What is meant by "dispersion," and when is it useful to measure it? Give a simple numerical example to illustrate your answer. (*Corporation of Accountants.*)

17. Consider the following series: 108, 122, 115, 133, 119, 164, 132, 169, 139, 155, 149, 157, 170, 149, 154, 101, 94, 85, 69, 136, 167, 165, 174, 160, 126, 198, 101, 102, 173, 162, 164, 160, 102, 72, 175, 161, 105, 115, 120, 136, 97, 87, 129, 184, 203.

Find the arithmetic mean, median, quartile, and quartile deviation of these numbers. (*Corporation of Accountants.*)

18. Prepare a diagram showing the practical utility of the Lorenz Curve. In what respects, if any, is it inferior to the coefficient of dispersion? (*London Association of Certified Accountants.*)

19. Define dispersion and state its usefulness to the statistician. (*London Association of Certified Accountants.*)

20. Calculate the standard deviation from the following data, using the short cut method:—

Exceeding.	Not Exceeding.	f.
7½	8½	2
8½	9½	4
9½	10½	5
10½	11½	7
11½	12½	9
12½	13½	3
13½	14½	2

(*London Association of Certified Accountants.*)

21. Define "Skewness" and show the formula of the coefficient relative to the first, the second, or third measure. (*London Association of Certified Accountants.*)

22. In connection with the science of statistics, and the application of statistical methods, what do you understand by the term Dispersion, and how is it measured?

How is the quartile coefficient of dispersion calculated?

State the advantages and/or disadvantages of employing this coefficient as a measure of dispersion. (*London Association of Certified Accountants.*)

23. To what types do the following formulæ relate:—

(a) $Z = l + c\left(\frac{f_2}{f_2 + f_1}\right)$

(b) $\sigma = \sqrt{\frac{\Sigma(m - x)^2 - n(a - x)^2}{n}}$.

Explain briefly each of the mathematical symbols shown in (a) and (b). (*London Association of Certified Accountants.*)

24. Set out the formulæ for measuring the first, second, and third moments of dispersion, and illustrate the practical utility of any one of such moments. (*London Association of Certified Accountants.*)

25. To what types do the following formulæ relate:—(a) (1) $\delta = \frac{\Sigma d}{n}$, (2) $\delta_M = \frac{\Sigma d_M}{n}$, (3) $\delta_Z = \frac{\Sigma d_Z}{n}$.

(b) Explain briefly each of the mathematical symbols in (a) 1, 2 and 3. (*London Association of Certified Accountants.*)

26. Draw a Lorenz Curve from the following data of the distribution of personal estates:—

Range of Estates.		No. of Estates.	Aggregate Capital value of Estates.
Exceeding.	Not Exceeding.		£ Millions.
—	£1,000	70,000	30
£1,000	5,000	30,000	80
5,000	10,000	7,000	50
10,000	20,000	4,000	60
20,000	100,000	3,000	150
100,000	500,000	400	80
500,000	1,000,000	20	10
1,000,000	—	10	40
		114,430	500

(*Incorporated Accountants* (*Final*).)

27. Compute the Quartile Coefficient of Dispersion of the following frequency table :—

Exceeding. *m.*	Not Exceeding. *m.*	*f.*	Exceeding. *m.*	Not Exceeding. *m.*	*f.*
4	5	2	9	10	15
5	6	4	10	11	12
6	7	9	11	12	7
7	8	7	12	13	3
8	9	11	13	14	1

(*Incorporated Accountants (Final).*)

28. What are the methods of computing and the purposes of the measures of dispersion? (*Building Societies Institute.*)

29. Find the mean deviation from the mean, and the mean deviation from the median, in the following list of marks :

100, 80, 78, 75, 74, 72, 70, 65, 63, 60, 60, 20.

(*Building Societies Institute.*)

30. The following table shows the age-distribution of Males in the Urban District of Bishop Auckland at the Census of 1931. You are required :—

(i) to draw a cumulative curve (*a*) upwards and (*b*) downwards graphing these figures;

(ii) from your curves to read off the median and quartile ages of the groups.

(iii) to give *one* measure of dispersion or deviation for the group.

Bishop Auckland U.D.C.

Age l.b.d.	No.	Age l.b.d.	No.
0–9 years	1,058	50–59 years	685
10–19 ,,	964	60–69 ,,	486
20–29 ,,	918	70–79 ,,	194
30–39 ,,	770	80 over	38
40–49 ,,	710		
		All ages	5,823

(*N.A.L.G.O.*)

CHAPTER XVII

CORRELATION. THE RELATIONSHIP BETWEEN SERIES OF DATA

THE discussions in previous chapters have dealt with various aspects of a single series of data, and in the consideration of deviation in the last chapter only the numerical measure of variations was taken into account. There are, however, many instances in which two or more groups or series of data are associated in some manner which results in some related movement when fluctuations occur. As one series fluctuates, another series shows a tendency to fluctuate also, either in the same or opposite direction. This co-variation is referred to as Correlation.

Correlation or Co-Variation is the relationship between two or more quantities that vary in sympathy, so that a movement in one is accompanied by a movement in the other in the same or an inverse direction.

Sales of some articles, like raincoats and umbrellas, tend to be greater on wet and cloudy days; there is causal relationship between prices and production; between sales, advertising, and profits; and between wages, production costs, and profits. These are examples of correlation between variables.

Direct Correlation exists when the variables tend to move in the same direction. It is sometimes called *positive correlation.*

Inverse Correlation exists when they tend to move in opposite senses. It is sometimes called *negative correlation.*

The Degree of Correlation may be analysed and measured, even though the cause and effect may not be determinable.

Statistical consideration of correlation includes: (*a*) the measurement of the amount of dependence between variables, and (*b*) the estimation of probable movements of one variable in terms of the other.

Perfect Correlation exists if the fluctuations in one series are precisely proportional with those in the other series. This rarely, if ever, occurs in business data.

Perfect correlation is indicated numerically as + 1 when direct, and − 1 when inverse; total absence of correlation is denoted by 0; and partial dependence by a coefficient lying between 0 and + 1 or 0 and − 1 as the case may be.

Linear and Non-Linear (Curvilinear) Correlation.—The distinction between these depends upon the constancy of ratio of change between two or more variables.

Linear correlation occurs when the amount of movement in one variable bears a constant ratio to the amount of change in the other. If the ratio of change is not constant the correlation is non-linear or curvilinear.

For example, assume that the output of men loading clay from a pit for a brickworks is doubled if the number of diggers is doubled, and varies in quantity as the number of men is reduced or increased, *there is linear correlation.*

Measurement of Correlation.—This may be effected by the following methods :—

(1) By constructing a graph or a scatter diagram.
(2) By showing the series in a correlation table.
(3) By computing a coefficient of correlation.

Graphic Method of Showing Correlation.—Correlation may be shown by ordinary historical curves, or by what is known as the scatter diagram. The two methods are illustrated in Figs. 76 and 77 based on the following hypothetical data.

Number of Sunny Days and Passengers Carried by the Gay-Time Motor Coach.

Month.	No. of Sunny Days.	No. of Passengers.	Month.	No. of Sunny Days.	No. of Passengers.
Jan. . .	10	100	July . .	4	50
Feb. . .	8	90	Aug. . .	8	100
Mar. . .	20	198	Sept. . .	20	180
Apl. . .	10	120	Oct. . .	6	80
May . .	21	178	Nov. . .	4	35
June . .	20	205	Dec. . .	4	42

Scatter Diagrams.—The line drawn in the scatter diagram in Fig. 77 is a *line of best fit* drawn simply by inspection so as to lie as close as possible to all the plotted points. A good method is to stretch a piece of thread and try out the best position for the line to take. If the points are too scattered to judge, owing to low correlation, there are formulæ in the form of a mathematical equation which can be used but these are outside the scope of this book.

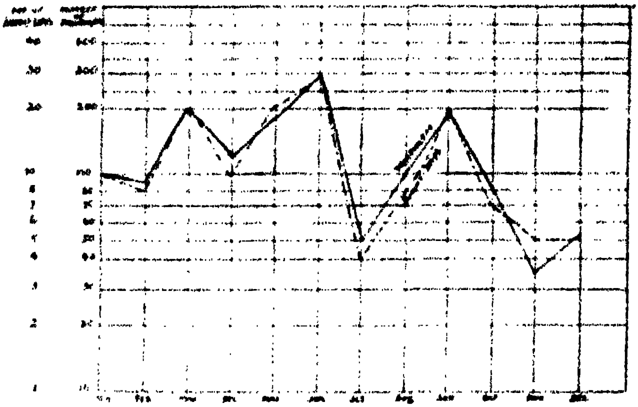


Fig. 76. High Correlation shown by Historical Curves.

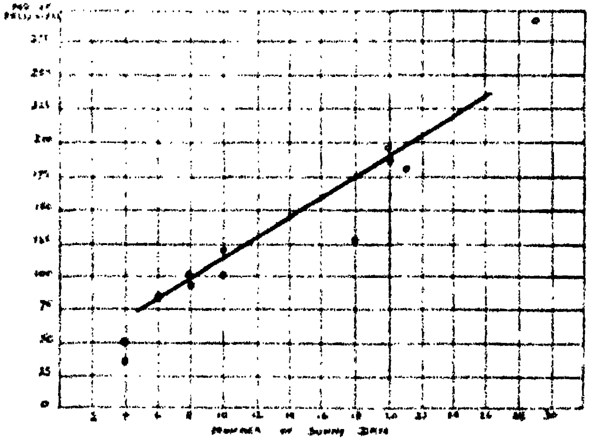


Fig. 77.—High Correlation shown by Scatter Diagram.

When there is little or no correlation, *i.e.* there is little or no tendency for the two series to vary in sympathy, the plotted points are scattered widely over the area of the graph, and no line of best fit is possible (Fig. 80).

When there is high direct correlation the line of best fit slopes upwards from left to right (Figs. 77 and 78); when there is high inverse correlation the line slopes downwards from left to right (Fig. 79).

Scatter Diagrams

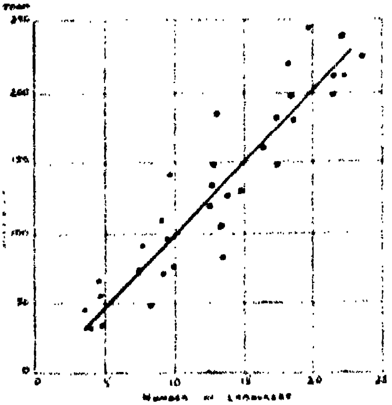


FIG. 78.—DIRECT CORRELATION.

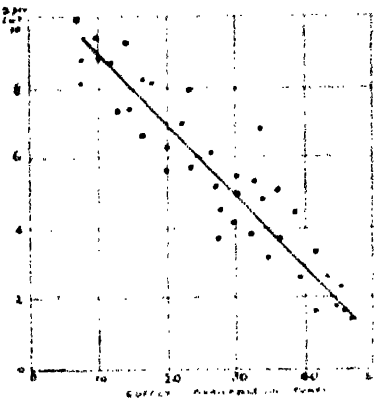


FIG. 79. INVERSE CORRELATION.

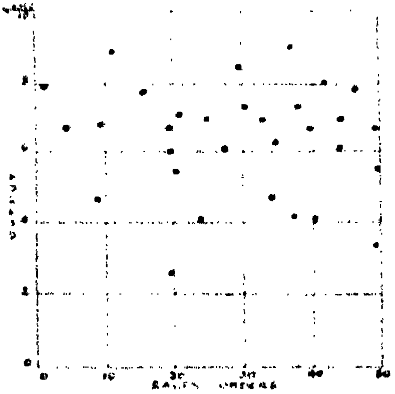


FIG. 80.—NO CORRELATION.

Choosing the Scale for a Scatter Diagram. The space allotted for the range of data of each series should be approximately equal, otherwise there may be difficulty in showing the correlation.

A simple method is to make the ratio between the scales inversely proportional to the ratio between the ranges of the data; *e.g.* range of data plotted on ox = 500; on oy = £100,000, then the scale would be 100,000 : 500 = 200 : 1, so that each square on ox would represent 1 unit, and each on oy would represent 200 units.

Historical Curves of two series plotted on the same graph will show correlation if it exists, because the movement of the two curves will be related. If they run parallel the correlation is direct or positive: if one curve moves similarly to another, but in the opposite vertical direction, the correlation is inverse or negative. A good method for revealing inverse correlation is to reverse the scale for one of the series: this makes the curves follow the same direction, and the degree of correlation is more easily inspected. (Compare Figs. 76 and 81.)

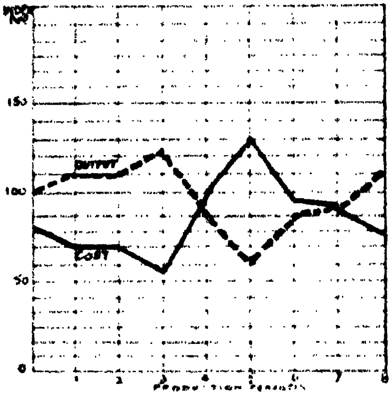


FIG. 81.—OUTPUT INDEX AND COSTS INDEX SHOWING INVERSE CORRELATION.

Another simple method, and the most practical one, is to bring the series together on a logarithmic graph.

Finally, trend and seasonal variations in correlation may be eliminated, by plotting standard deviations of the data. A lesser degree of exactitude is obtained if the interquartile deviations or average deviations are used.

In plotting data, the causal or independent variable should be plotted along the x axis, as this is the usual procedure, and it is better when estimations have to be made from the curve plotted.

The diagram in Fig. 82 is an interesting application; it shows that it is cheaper to make the components by hand up to a quantity of 175 gross, but greater quantities can be produced more economically by machinery.

Estimating from Scatter Diagrams.—When a scatter diagram has been constructed and a line of best fit inserted, if the value of one of the series is known and not the other, the latter can be estimated by finding the point where a line drawn from the axis towards the line of best fit intersects that line. If from this point of intersection, say by a line from the *x* axis, a dotted line is drawn parallel with that axis, the desired estimate can be read on the *y* axis at the point where the dotted line intersects it. In Fig. 79 when the crops available on the market are 2,500 tons it is probable the price would be 6s. per cwt.

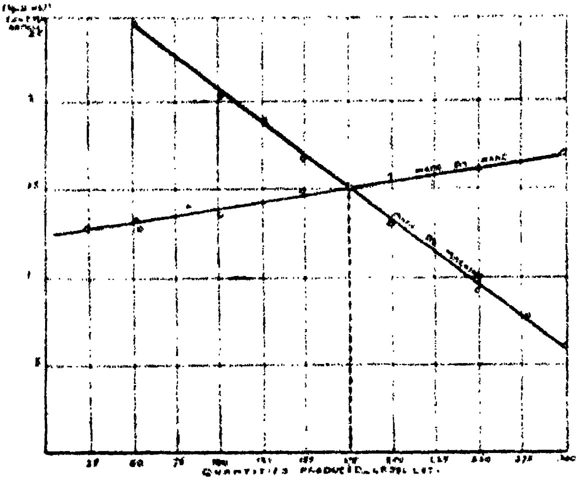


FIG. 82.—SCATTER DIAGRAM SHOWING THE POINT AT WHICH IT IS CHEAPER TO MAKE BY MACHINERY.

Correlation Tables.—By constructing a table instead of a scatter diagram, the data will be found to show an arrangement of the figures which corresponds with the arrangement of the dots in the scatter diagram. It gives the added advantage that the number of cases within each class interval can be seen. A simple example illustrating the construction is given in Fig. 83. If correlation were low or non-existent, the figures would be well scattered in each of the columns instead of lying in an ascending or descending band across the table, thus corresponding with the scatter diagram as illustrated in Fig. 78.

Measurement by Coefficient of Correlation.—There are many formulæ for expressing the degree of correlation between two or more series of quantitative data, but the most commonly used is the sum-product method of Karl Pearson, by which can be calculated what is known as the Pearsonian coefficient of correlation.

Weekly Output of Clay in Relation to the Number of Diggers.

Output (tons).	Men Engaged.				
	1–5.	5–10.	10–15.	15–20.	20–25.
200–250				2	4
150–200			1	3	1
100–150		2	6	1	
50–100	2	5	1		
1– 50	3	1			

FIG. 83.—CORRELATION TABLE SHOWING HIGH DIRECT CORRELATION.

This formula is used when the series tend to approximate to a straight line when plotted in a scatter diagram, and is as follows :—

$$r = \frac{\Sigma xy}{n\sigma X\sigma Y}.\quad \text{Sometimes written } \frac{\Sigma xy}{n\sigma_1\sigma_2}.$$

r = usual letter to signify coefficient of correlation.
x = deviations from the mean of X series.
y = ,, ,, ,, of Y series.
n = number of items.
$\left.\begin{matrix}\sigma X \text{ and } \sigma Y\\ \text{or } \sigma_1 \text{ and } \sigma_2\end{matrix}\right\}$ = the standard deviations of the two series.

The use of the formula may be shown in connection with the data in Fig. 84 :—

To Arrive at Coefficient of Correlation, tabulate the two series as shown in Fig. 84*a*, total the items and find the average (arithmetic) for each series (viz. of X = 15 ; of Y = 34).

Subtract this mean from each item, and enter in columns x and y respectively as in Fig. 84*a*, inserting proper signs. Square these deviations, and enter in two more columns x^2 and y^2 respectively, *Number and Value of Castings Produced during* 10 *Years* 19.., *to* 19....

Year.	Number (000's).	Value (£000's).
1	11	21
2	9	17
3	11	20
4	17	38
5	18	38
6	19	40
7	19	42
8	18	46
9	16	40
10	12	38
Total	150	340

FIG. 84.—DATA FOR TABLE, FIG. 84*a*.

Year.	Numbers (X).			Value (Y).			Product of Deviations (xy).
	000's.	Deviations from Average (x).	Deviations Squared (x^2).	£000's.	Deviations from Average (y).	Deviations Squared y^2.	
1	11	− 4	16	21	− 13	169	+ 52
2	9	− 6	36	17	− 17	289	+ 102
3	11	− 4	16	20	− 14	196	+ 56
4	17	+ 2	4	38	+ 4	16	+ 8
5	18	+ 3	9	38	+ 4	16	+ 12
6	19	+ 4	16	40	+ 6	36	+ 24
7	19	+ 4	16	42	+ 8	64	+ 32
8	18	+ 3	9	46	+ 12	144	+ 36
9	16	+ 1	1	40	+ 6	36	+ 6
10	12	− 3	9	38	+ 4	16	− 12
Total	150		132	340		982	316
Average	15			34			

$$\text{Standard Deviation } X = \sqrt{\frac{132}{10}} = 3\cdot63.$$

$$\text{Standard Deviation } Y = \sqrt{\frac{982}{10}} = 9\cdot91.$$

$$\text{Coefficient of Correlation} = \frac{316}{3\cdot63 \times 9\cdot91 \times 10} = 0\cdot878.$$

FIG. 84*a*.—CALCULATION OF COEFFICIENT OF CORRELATION OF DATA IN FIG. 84.

Multiply the deviations x and y for each year, and enter in final column; total this column observing signs. This total (316) is the numerator (Σxy) in the formula.

Then find the standard deviation of X thus :—

$$\sqrt{\frac{\text{Total of column } x^2}{\text{No. of items}}} = \sqrt{\frac{132}{10}} = 3{\cdot}63$$

and the standard deviation of Y :—

$$\sqrt{\frac{\text{Total of column } Y^2}{\text{No. of items}}} = \sqrt{\frac{982}{10}} = 9{\cdot}91.$$

Now multiply together the standard deviation of X and Y and the number of items, viz :

$3{\cdot}63 \times 9{\cdot}91 \times 10 = 359{\cdot}7$ (denominator ($n\sigma X\sigma Y$) of formula).

The coefficient of correlation is then found by dividing the total of the final column (316) by last figure found (359·7)

$$\therefore \frac{316}{359{\cdot}7} = 0{\cdot}878.$$

Interpretation of the Coefficient.—The coefficient of correlation, as stated in a previous paragraph, must lie between + 1 (perfect direct correlation) and − 1 (perfect inverse correlation). Complete absence of correlation is represented by 0.

When the coefficient is 0·95 or over the degree of correlation is high, and from a point in the series of one the corresponding value of the item in the other series may be accurately estimated.

When the coefficient is between 0·75 and 0·90, a similar estimate will be a fairly reasonable one. But a lower coefficient than those mentioned cannot be used for practical estimating.

It should be remembered that the coefficient calculated by the above formula is only reliable for a straight-line distribution in a scatter diagram, and further, this mathematical calculation of the degree of relationship should always be used with great caution when being applied to practical inquiries. Usually the rougher graphical method will give sufficient measure of relationship for practical use.

Probable Error of the Coefficient of Correlation.—This defines the limit above and below the size of the coefficient determined within which there is an equal chance that any coefficient of correlation similarly calculated from other samples will fall.

It is calculated by the formula :—

$$\text{Probable Error of } r = \frac{0{\cdot}6745(1 - r^2)}{\sqrt{n}}.$$

Thus suppose the coefficient (r) is 0·90 as calculated in the manner described above from 16 pairs of items,

$$\text{P.E. of } r = 0{\cdot}6745\,\frac{(1 - 0{\cdot}90^2)}{\sqrt{16}} = \frac{0{\cdot}1281}{4} = 0{\cdot}03,$$

i.e. the coefficient of correlation may vary from 0·90 + 0·03 and 0·90 − 0·03, *i.e.* between 0·93 and 0·87.

Hence, if it is desired to state the coefficient of correlation and its probable error, we could write it as

$$r \pm \frac{0{\cdot}6745\,(1 - r^2)}{\sqrt{n}}.$$

If r is less than the probable error, there is no correlation.

If r is more than six times the size of the probable error, it is usually regarded as satisfactory evidence of correlation.

The formula given supposes unbiassed choice of samples of equal size, and deals with error arising from the limited number of items used (not from the size of the items or groups). As the conditions of unbiassed data and independence of individual items (especially in a time series of business data) are seldom present in business data, the calculation of probable error by the formula is not useful or practicable.

Although of little use in business, the following formulæ are given as other measures of probable error not connected with correlation, but with random sampling :—

Probable error of a distribution

$$= 0{\cdot}6745 \times \text{Standard Deviation, } i.e.\ 0{\cdot}6745 \times \sigma.$$

Probable error of the mean

$$= \frac{0{\cdot}6745 \times \text{Standard Deviation}}{\sqrt{n}},\ i.e.\ \frac{0{\cdot}6745 \times \sigma}{\sqrt{n}}.$$

Probable error of the standard deviation

$$= \frac{0{\cdot}6745 \times \text{Standard Deviation}}{\sqrt{2n}},\ i.e.\ \frac{0{\cdot}6745 \times \sigma}{\sqrt{2n}}.$$

The Ratio of Variation between two variables, and the use of the Galton Graph for its determination are described on page 187.

Lag and its Use in Statistics.—It is not uncommon for certain factors to affect the movement of other factors, but only after an interval of time has elapsed. Correlation exists, but in a graph one curve lags behind the other. Speculation may move ahead of general business; advertising and sales organisation expenditure is often ahead of sales expansion: American statistics have shown that prosperous business conditions led to increased immigration in the succeeding year. If lag is known to exist, it is better to allow for it by lagging the "effect" curve to make it comparable with the "cause" curve when making comparative historigrams.

Publicity Expenditure and Sales, 19..

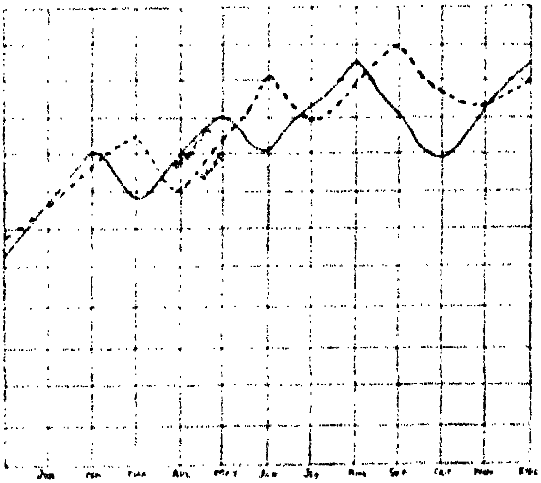


FIG. 85.—HISTORIGRAMS DRAWN TO DETERMINE LAG.

The most convenient way of determining the lag period is to draw two separate graphs, using the same scale, one on semi-translucent paper, so that when one is superimposed it can be moved along in the known direction until the best fit is found. The time interval can then be read and allowed for.

Alternatively two curves can be plotted on one graph, as in Fig. 85, in which it will be seen the lag of sales behind publicity expenditure is about one month, and when finding the coefficient of correlation the figures for publicity for January should be paired with those for February sales; February publicity with March sales, and so on.

Another method is to prepare "scatter" diagrams for the related data, pairing the figures with varying intervals, *e.g.* publicity for January with sales for February; then January and March, and so on, until the plotted diagram shows the best indication of correlation.

Another method, giving closer measurement, is to plot, on a natural scale graph, the coefficients of correlation calculated with different pairings of the data. Then draw a smoothed curve. The highest point on this curve is the amount of the lag, and when that is allowed for, correlation will be highest.

QUESTIONS

1. Calculate the coefficient of correlation for the following ages of husband and wife:—

Age of Husband.	Age of Wife.	Age of Husband.	Age of Wife.
23	18	30	29
27	20	31	27
28	22	33	29
28	27	35	28
29	21	36	29

(*Incorporated Accountants (Final)*.)

2. The table shows the distribution of wage earners in a company by weekly earnings.

July 1933.

No. of Wage-earners.	Weekly Earnings.	
	Over.	Not Exceeding.
250	10*s*.	20*s*.
700	20*s*.	30*s*.
1200	30*s*.	40*s*.
1853	40*s*.	50*s*.
2700	50*s*.	60*s*.
3048	60*s*.	70*s*.
1963	70*s*.	80*s*.
750	80*s*.	90*s*.
200	90*s*.	100*s*.

From the above table obtain by graphical methods: (*a*) the median, (*b*) the upper and lower quartiles. From these calculate the quartile coefficient of dispersion. (*London Chamber of Commerce.*)

3. The price index numbers of ten foodstuffs for two years 1913 and 1930 are :—

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
1913 . . .	57	65	66	70	72	73	67	81	92	97
1930 . . .	77	91	84	91	95	112	95	143	120	132

Find a measure of the correlation between these two sets of indices. (*London Chamber of Commerce.*)

4. Explain briefly to what the following formula relates and state its practical utility in statistics :—

$$\frac{0{\cdot}6745(1 - r^2)}{\sqrt{n}}$$

(*London Association of Certified Accountants.*)

5. It has been stated that an increase in the cost of production causes the price of a commodity to rise, and that the resulting fall in the demand will lower the price again and cause it to return to its former level.

Prepare a diagram illustrating this statement. (*London Association of Certified Accountants.*)

6. Compute the quartile coefficient of dispersion of the following frequency table :

Exceeding.	Not Exceeding.	£.
7½	8½	2
8½	9½	4
9½	10½	6
10½	11½	7
11½	12½	9
12½	13½	3
13½	14½	1

7. (*a*) What use is made of the Lag in the study of Statistics?

(*b*) Interpret the formula :—

$$r = \frac{\Sigma(xy)}{n\,\sigma_1\,\sigma_2}$$

(*London Association of Certified Accountants.*

8. Explain what is meant by a coefficient of correlation and what it measures. (*Union of Lancashire and Cheshire Institutes.*)

9. The prices, in pence per pound, of two grades of cotton for twenty successive weeks during 1933 were as follows :

Mid-American .	5·02	4·98	4·85	4·79	5·09	5·15	5·06	5·16	5·35	5·36
No. 1 Oomra .	4·20	4·17	4·09	3·97	4·27	4·24	3·92	3·86	4·03	4·03
Mid-American .	5·41	5·73	5·87	6·02	5·99	6·30	6·37	6·24	6·21	6·59
No. 1 Oomra .	4·17	4·39	4·48	4·66	4·50	4·88	4·93	4·81	4·77	5·15

Find approximately the coefficient of correlation between the two sets of prices. (*Union of Lancashire and Cheshire Institutes.*)

10. Examine the influence of the numerator in the Karl Pearson coefficient of correlation. (*Incorporated Accountants* (*Final*).)

11. Compute the coefficient of correlation for the following series of ages of husband and wife :—

Subject. *m.* Age of husband.	Relative. *m.* Age of wife.	Subject. *m.* Age of husband.	Relative. *m.* Age of wife.
27	22	30	29
28	26	31	32
29	24	31	26
29	21	32	25
30	25	33	30

(*Incorporated Accountants* (*Final*).)

12. How would you test whether variations in the price of primary commodities exported from the countries of the British Empire overseas were a guide to the changes in the volume of exports from the United Kingdom to those countries? (*Incorporated Accountants* (*Final*).)

13. Define correlation, and explain the meaning of positive correlation and negative correlation. (*Building Societies Institute.*)

CHAPTER XVIII

TIME VARIATIONS AND TRENDS

Comparisons of Business Facts and Tendencies.—It is recognised that comparative statistics of business are one of the most effective methods of analysis for management purposes.

Statements and graphs showing data for a current period, with corresponding data for some preceding period, *e.g.* last month, corresponding month last year, etc., are now very commonly used by alert business executives.

Further valuable information may be afforded by preparing tables or graphs showing fluctuations of a period of time, especially over a series of months, and for some purposes, over a series of years. This aspect of data will now be briefly considered.

Time Variations may reveal four types of movement :—

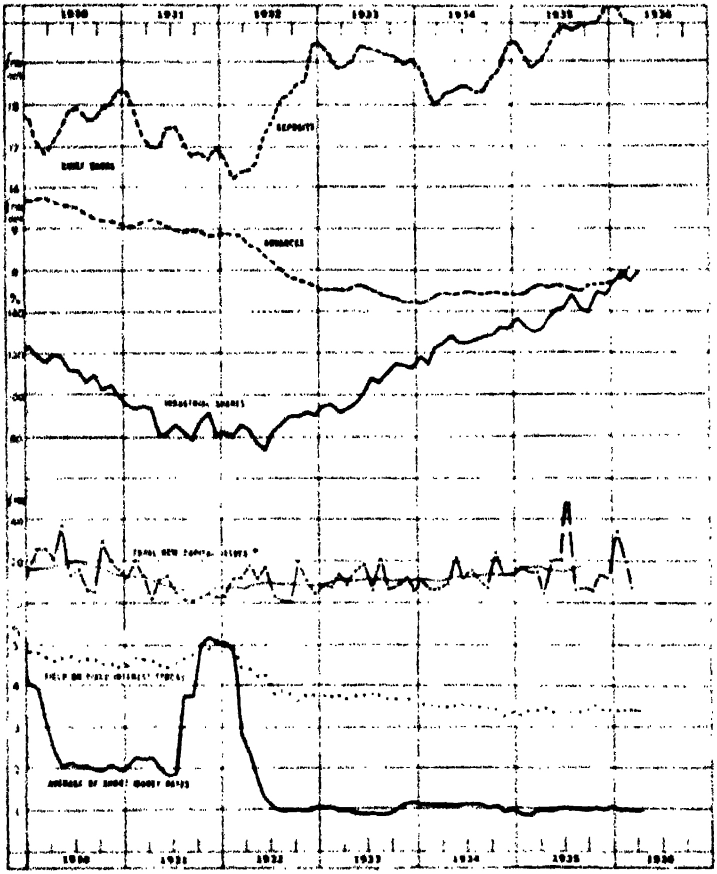
(1) *Short Time or Seasonal Fluctuations.*—These are frequently of a regular periodic type, movements rising and falling hourly, daily, weekly, seasonally, etc., according to the nature of the business. Seasonal oscillations are a marked feature of many trades and industries. (See Fig. 94, showing regular seasonal fluctuations in coal production; also the fairly regular quarterly fluctuations in bank deposits in Fig. 86.)

(2) *Cyclical Fluctuations* are longer-termed movements of a periodic type, but which recur at intervals of more than a year. Within these major cyclical movements the more frequent seasonal fluctuations also occur, just as small waves keep breaking on the undulating swell of a rough sea. (See top curve of Fig. 86 representing Bank Deposits.)

(3) *Long-Time or Secular Movements* representing the long-period trend over a number of years. (See the superimposed moving average of new capital issues in Fig. 86.) It is not unusual to have seasonal fluctuations of data imposed on a cyclical rise and fall over two, three or more years, with the whole of the data showing a long-period rising trend.

(4) *Irregular Fluctuations*, often of an unexpected and violent type arising from some exceptional occurrence like war, fire or flood, or other derangement of normal economic conditions. (See bottom curve of Fig. 86, showing sharp rise in short money rates in 1931.) These irregular and erratic movements are often not separable from the business cycle variations.

FINANCE.



† 12-months moving average superimposed.

FIG. 86.—CURVES REPRESENTING VARIOUS TYPES OF TIME VARIATIONS.

The four types of time variations are illustrated in Fig. 86,* the data for which appear in Fig. 97.

The Value of Analysing Time Variations cannot be over-estimated. It is valuable to know, not only the weekly or other fluctuations of the current time, but also what is the general movement of affairs over a longer period. This is necessary when weighing up the question of ordering or contracting for new stock, or of embarking on new capital expenditure on plant or other fixed assets. Large stocks acquired when prices are soaring and capital installations when trade is booming have often resulted in heavy loss when conditions turn inversely. A skilful statistical analysis, closely and regularly watched, will often mitigate the difficulties when the turning point arrives.

The astute business man will study the causes of developments which have been taking place, and review what factors are operating to-day, so as to attempt an estimation of what all these factors are tending towards in the immediate future. To forecast the future is never certain, but judgments will be better guided if suitably prepared statistics are available and interpreted with reason. Graphs are particularly useful for showing business time-series of data.

The Basic Long-Period Trend is the tendency of various business values to increase or decrease over a period of years, as described above. This tendency goes on regardless of seasonal, short cyclical and irregular changes; and the value of ascertaining its movement is that as the direction does not change rapidly, it portrays probable trend for a year or more ahead, always taking care to bear in mind current factors likely to cause temporary variation, such as that of the business cycle. The business cycle is a swinging movement which has not the regularity of seasonal movements.

Three commonly occurring types of trend are :—

(*a*) *Arithmetic Progression*—a regular increase or decrease by the same *amount* each year. On a graph it is a straight line.

(*b*) *Geometric Progression*—a regular increase or decrease by the same *percentage or rate* each year. On a graph on a logarithmic chart it is a straight line.

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* Reproduced by permission from the *Monthly Bulletin*, April 1936, of the London and Cambridge Economic Service.

(c) "*S-Curve*" *Progression*—representing the first slow, then more rapid growth of the business up to a maturity point, after which any increase is small and slow. A graph of this type is given in Fig. 87. It is sometimes called a logistic curve.

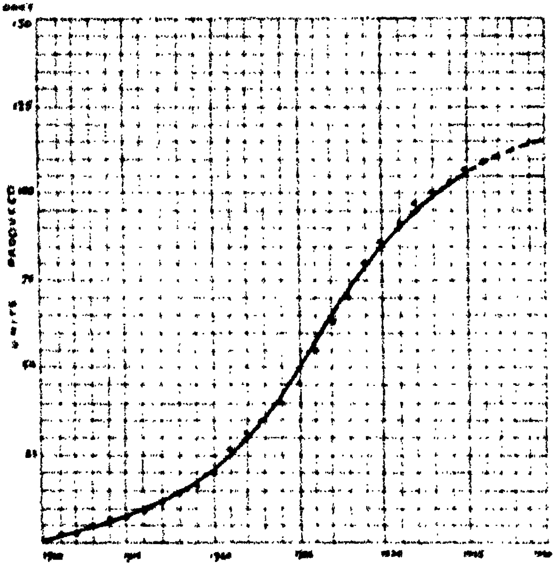

FIG. 87.—S-CURVE TREND OF PRODUCTION QUANTITIES BY A MANUFACTURING COMPANY.

Establishing the Trend Line.—A trend line is not regarded as very reliable for periods of less than ten years, and two things to guard against are :—

(1) Isolated extreme items, like a very exceptionally high year of trade due to a local non-recurring circumstance, *e.g.* an international exhibition in the locality ; an unusual influx of a temporary labour supply on a large constructional contract in the district ; a great fire, etc.

(2) Choice of wrong periods. Sometimes a pronounced cyclical movement may give a wrong impression of trend.

Various methods may be employed for fitting the secular or long-period basic trend—

(a) *Freehand Curves.*—These are very useful, as they may be guided to some extent by judgment. In case of doubtful direction it is a good plan to draw two possible extreme curves, and then insert the probable centre one in the most likely representative direction.

(b) *The Method of Semi-Averages.*—The series of data is divided into halves, an average taken of each half, and these semi-averages are plotted on the middle ordinate of each of the two periods. A line drawn through these two points gives the trend.

The value of the method is:—(i) that an exceptionally high or low year which is obviously unrepresentative may be omitted from the averaging. (ii) In an irregular series, two or more groups of years can be averaged, each being plotted separately, so that abrupt changes in trend can be shown.

(c) *Moving Average Method.*—This is a more precise method. Items are averaged in groups of three, five, seven or other suitable number, and the average so obtained is used as the figure for the centre year of each group, thus:—1929, 1930, 1931 data averaged gives the figure for 1930; 1930, 1931, 1932 averaged gives the figure for 1931, and so on for the whole series. (See example in Fig. 91, column (*c*), a moving average based on twelve monthly totals.) In selecting the number of items to be averaged regard must be had to which grouping will best eliminate the effects of irregular values; this is usually a period equal to the period of the cycle to be removed, or to an average of the cycles if the cycle is irregular.

If an even number of years be used, the average will be that of the two centre items, and the point will be plotted accordingly; or, the moving averages for those *two* years may be averaged and plotted exactly on the years. (See Fig. 91, column (*d*).)

A disadvantage is that it cannot be kept close up to date, although for long period trend this is not a serious handicap.

The effect of plotting the moving average of a series of data is shown in Fig. 37, and in Fig. 86 (the curve representing "New Capital Issues").

(d) *The Method of Least Squares.*—Although this is the most representative method, it is rather involved for practical use. The procedure is :—

(i) Find the arithmetic average of the items; this is the middle point of the line representing the trend.

(ii) Find the centre of the series in point of time.

(iii) Subtract each year from the mid-year to find the *time* deviation from the latter. (See column (*c*), in Fig. 88.) Those before the mid-point are minus deviations, those after are plus.

(iv) Square these deviations and total the figures so found.

(v) Multiply the value of the items (column (*b*)) by the deviations (those in column (*c*)), and total the products, taking notice of the signs.

(vi) Divide the total found in (v) by the total found in (iv). The result is the average amount of increase in the trend year by year, thus giving the slope of the line of least squares which is the line of best fit. This increment is used thus: For the number of years above or below the mid-year that a given year stands in relation to the mid-year add or subtract that many increments. Thus 1932 to 1936, mid-year is 1934; say the increment is £250, the increment for 1935 is 1 × £250 = £250 above the average value for the series of data, viz. £3000; for 1936 it is 2 × £250 = £500 above the average; for 1932 it is 2 × 250 = £500 below the average and so on, as shown in column (*f*), Fig. 88. The values in column (*f*) when plotted show the trend line. (See Fig. 89.)

N.B.—When, instead of a progressive amount of increase (or decrease), there is a constant percentage increase each year, the trend can be shown by the above method, substituting logarithms for the actual data. Plotted on a semi-logarithmic chart, the result is a logarithmic straight line of trend, sometimes called a compound-interest curve. This method is useful for estimating increases of population as the population increases according to the compound interest law.

Seasonal Variation.—When actual data, say of sales, are plotted on a natural scale graph, the seasonal variation is easily seen, and usually if each year's sales totals have increased (as they will in a progressive business), the seasonal variation movements are more pronounced. Plot the same data on a logarithmic scale and the variations will usually appear as regular waves throughout the series. Seasonal variation is usually sufficiently indicated by monthly data.

Measurement of Seasonal Variation.—The three most practical methods are :—

Year. (a)	Sales. (b)	Time Deviation from Middle Year. (c)	Deviations of Col. (c) Squared. (d)	Product Cols. (b) × (c). (e)	Trend Ordinates. (f)
	£				
1932	2,250	− 2	4	− 4500	2500
1933	2,800	− 1	1	− 2800	2750
1934	3,900	0	0	0	3000
1935	2,300	+ 1	1	+ 2300	3250
1936	3,750	+ 2	4	+ 7500	3500
	15,000		10	+ 2500	

Average £3000 (= trend point value for 1934).

Year to year trend increase in sales $= \frac{\text{Col. }(e)}{\text{Col. }(d)} = \frac{2500}{10} = £250$. This amount is added or deducted from £3000 according to the time deviations given in column (c) (see column (f)).

The values in column (f) are plotted on the graph in Fig. 89.

FIG. 88.—THE LINE OF BEST FIT SHOWING TREND FORMED BY THE METHOD OF LEAST SQUARES.

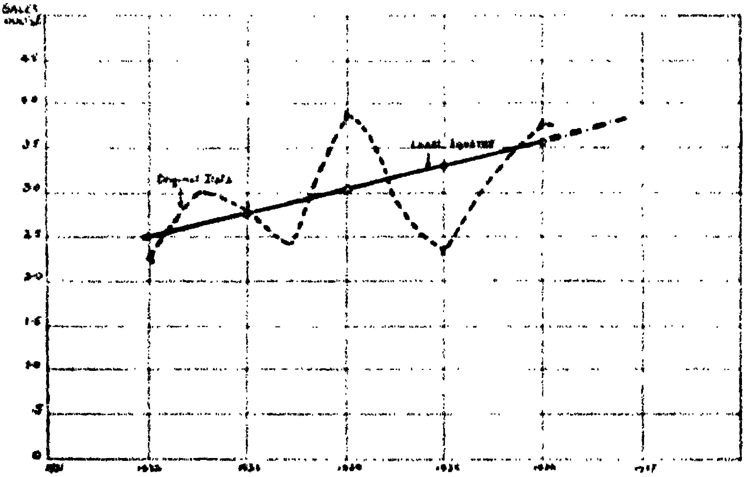

FIG. 89.—TREND BY PLOTTING VALUES FOUND BY METHOD OF LEAST SQUARES FROM DATA IN FIG. 88.

(1) Use of monthly averages to form a seasonal index.

(2) Use of moving averages to form a seasonal index.

(3) Method of link relatives, a method which is too complicated for ordinary use and will not be described.

The advantage of measuring seasonal variations is that the causes may be inquired into and the possibility of changing them (*e.g.* by advertising) may be considered.

Calculation of Seasonal Variation by Monthly Averages.—The procedure is :—

(i) Find the sum of the totals (say sales) for the Januarys, Februarys, etc., for the years under review (Fig. 90, column (*f*)), and divide by the number of totals used to obtain an average for January, February, etc. (column (*g*)).

(ii) Find the average of the monthly totals. This can be based on the sum of the monthly totals (column (*f*)), or on the sum of the monthly averages (column (*g*)). Both are given in Fig. 90 to show that it does not matter which are used for the purpose.

Mandarin Limited.

Months.	Sales.				Total of Four Years.	Four-Years' Average.	Percentage of Monthly Total to Average Monthly Total.
	1933.	1934.	1935.	1936.			
(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)
	£	£	£	£	£	£	£
Jan. .	260	266	270	276	1,072	268	114·5
Feb. .	230	238	242	250	960	240	102·6
March .	195	202	228	234	859	215	91·9
April .	190	196	199	208	793	198	84·6
May .	200	210	216	222	848	212	90·6
June .	210	219	225	230	884	221	94·4
July .	155	159	176	175	665	166	70·9
Aug.	190	196	198	200	784	196	83·8
Sept. .	220	226	234	240	920	230	98·3
Oct. .	240	242	250	260	992	248	106·0
Nov. .	294	298	312	320	1,224	306	130·8
Dec. .	296	300	310	325	1,231	308	131·6
Total .					11,232	2808	1200·0
Average					936	234	100·0

Fig. 90.—Calculation of Seasonal Variation Index (Column (*h*)).

Mandarin Limited

Years and Months. (*a*)		Actual Sales. (*b*)	Twelve Months Moving Average. (*c*)	Two Months' Moving Average of Col. (*c*). (*d*)	Sales (Col. (*b*)) as Percentages of Moving Average. (*e*)
1937	Jan. .	266			
	Feb. .	238			
	March .	202			
	April .	196			
	May .	210			
	June .	219	233·5		
	July .	209	233·8	233·6	89·4
	Aug. .	196	234·2	234·0	83·7
	Sept. .	226	236·3	235·2	96·0
	Oct. .	242	236·6	236·4	102·4
	Nov. .	298	237·1	236·8	125·8
	Dec. .	300	237·7	237·4	126·4
1938	Jan. .	270	239·0	238·4	113·3
	Feb. .	242	239·1	239·0	101·3
	March .	228	239·8	239·4	95·2
	April .	199	240·5	240·1	82·9
	May .	216	241·7	241·1	89·6
	June .	225	242·5	242·1	92·9
	July .	226	243·0	242·7	93·1
	Aug. .	198	243·7	243·3	81·4
	Sept. .	234	244·2	243·9	96·0
	Oct. .	250	244·9	244·5	102·2
	Nov. .	312	245·4	245·1	127·3
	Dec. .	310	245·8	245·6	126·2
1939	Jan. .	276	245·8	245·8	112·3
	Feb. .	250	246·0	245·9	101·7
	March .	234	246·4	246·2	95·0
	April .	208	247·3	246·8	84·3
	May .	222	248·0	247·6	89·7
	June .	230	249·2	248·6	92·5
	July .	225			
	Aug. .	200			
	Sept. .	240			
	Oct. .	260			
	Nov. .	320			
	Dec. .	325			

FIG. 91.—TABLE OF MOVING AVERAGES, BASED ON ACTUAL SALES IN COLUMN (*b*).

The percentages in column (*e*) above calculated thus: July $\frac{209}{233 \cdot 6} \times 100 = 89 \cdot 4$, etc.

(iii) Find the percentage that each monthly average or total bears to the average of the totals, or to the total of monthly totals (column (*h*)), *e.g.* January (column (*g*)) :—

$$\frac{\text{Average Total for January}}{\text{Average of Total based on 12 months average}} \times 100$$

$$\text{hence } \frac{268 \times 100}{234} = 114{\cdot}5 \text{ (as in column } (h)).$$

In practice a longer period than the four years dealt with in Fig. 90 should be used because of the influence of cyclical factors.

Mandarin Limited

Sales 1937–1939

Months. (*a*)	1937. (*b*)	1938. (*c*)	1939. (*d*)	Average (Cols. (*b*), (*c*), (*d*)). (*e*)	Seasonal Index. (*f*)
Jan. . .		113·3	112·5	112·0	113·0
Feb. . .		101·3	101·7	101·5	101·6
March .		95·2	91·0	93·1	93·2
April .		82·9	84·3	83·6	83·7
May . .		89·6	89·7	89·6	89·7
June .		92·9	92·6	92·7	92·8
July . .	89·4	93·1		91·2	91·3
Aug. .	83·7	81·4		82·5	82·6
Sept. .	96·9	96·0		96·4	96·5
Oct. . .	102·4	102·2		102·3	102·4
Nov. .	125·8	127·3		126·5	126·6
Dec. . .	126·4	126·4		126·4	126·5
				12)1198·7	—
				99·9	

FIG. 92. SEASONAL INDEX BASED ON MOVING AVERAGE.
Only three years used for convenience of space.

Calculation of Seasonal Variation by Moving Averages.—The procedure is :—

(i) Find moving averages. (See Fig. 91, columns (*c*) and (*d*).)

(ii) Express actual data as percentages, item by item each year, of the moving averages (see Fig. 91, column (*e*)), and arrange in columns as in Fig. 92, columns (*b*), (*c*), and (*d*).

(iii) Average percentages in (ii) month by month for the four (years or other number of years). (See Fig. 92, column (*e*).)

(iv) Convert these averages in (iii) to relatives using the average of these monthly averages as base. For example, January in Fig. 92. column (*e*) : $\frac{112\cdot9}{99\cdot9} \times 100 = 113$; December $\frac{126\cdot4}{99\cdot9} \times 100 = 126\cdot5$, as shown in column (*f*).

The Elimination of Seasonal Effects is advantageous when it is desired to observe the trend of business data, as the trend is then much easier to see. Two methods may be used :—

(i) Compute the moving average for a period of one year, in the manner already described, or

(ii) Divide the actual data (of sales, production, etc.) by the seasonal index. For example, by dividing the monthly sales in column (*b*) of Fig. 91 by the corresponding monthly index in Fig. 92 (column (*f*)) the data will be corrected from the influence of seasonal variations.

Business Cycles, or, in other words, the recurrence of periods of good and bad times over a number of years, are most conveniently reviewed in relation to the "normal," *i.e.* in relation to the long time trend as affected by the usually inevitable seasonal variations.

Without the interference of the business cycle the normal, as just defined, would be the course followed by most well-managed businesses. If, therefore, the influence of trend and seasonal variation were removed, the normal course would be represented on a graph by a straight line parallel to the base line of the graph, and the cyclical and erratic movements would rise above and fall below that normal line.

To Show Cyclical Movements from the Normal the following methods may be used :—

Method 1.

(i) Find the trend (say by the moving average or method of least squares already described), and the seasonal index. Say these are respectively sales £1000 per month, seasonal index 80 per cent.

(ii) Then the normal is 80 per cent. of £1000 per month, viz. £800.

(iii) Deduct this "normal" from the actual sales for each month. The differences (plus or minus) are the effect of the cyclical movements, and will, if plotted on a graph, rise and fall above and under the horizontal " normal " line.

Method 2.

(i) Find the normal data (say sales) by multiplying the trend and seasonal index as before.

(ii) Express actual data as percentages of normal.

(iii) The deduction of the normal (100) from the percentages gives the deviations required, these being the result of cyclical movements without the effects of trend and seasonal variation.

Method 3, which is less accurate :—

(i) Reduce actual data to percentages of trend.

(ii) Subtract seasonal percentages from these.

This gives the percentage deviations from the normal. The most effective is Method 2, which shows the cyclical movements as percentages of the normal.

An Important Use of Analysis of Time Series is the assistance afforded in preparing business budgets and in attempting to forecast business probabilities. These problems will be dealt with in the next chapter.

QUESTIONS

1. Graph the short-time oscillations of the following price indices, using a half-yearly base :—

Month.	Index.	Month.	Index.
1	80	16	96
2	82	17	98
3	86	18	106
4	91	19	114
5	83	20	112
6	85	21	109
7	89	22	106
8	96	23	112
9	93	24	120
10	90	25	118
11	91	26	112
12	94	27	110
13	100	28	107
14	105	29	113
15	102	30	115

(*Incorporated Accountants* (*Final*).)

2. Draw a graph based on deviations from the arithmetic means, comparing the movement of the following series of average wages and average cost of living indices.

Year.	Weekly Average.	Average Index of Cost of Living.
1915	42·00	135·0
1916	44·58	165·0
1917	53·08	187·5
1918	69·91	220·0
1919	80·91	225·0
1920	100·83	265·0
1921	88·33	192·0
1922	71·33	178·0
1923	69·00	177·0
1924	73·42	180·0
1925	73·67	175·0
1926	73·83	175·0
1927	74·08	168·0
1928	72·50	167·0
1929	72·33	166·0

(*Incorporated Accountants* (*Final*).)

3. Show graphically the long-period trend and the short-period oscillations of the following data. Use a five-year cycle.

1880	80	1885	85	1890	91
1881	82	1886	89	1891	94
1882	86	1887	96	1892	100
1883	91	1888	93	1893	105
1884	83	1889	90	1894	102

(*Incorporated Accountants* (*Final*).)

4. How would you make a comparative study of :—(*a*) the long-time trend; and (*b*) the short-time fluctuations in two series of data expressed respectively in (*a*) tons, and (*b*) £ sterling? (*Incorporated Accountants* (*Final*).)

5. Define a moving average and state its various uses. What should determine the length of the average? (*Incorporated Accountants* (*Final*).)

6. The population of a country increases according to the compound interest law. In 1831 and 1931 it was 2,000,000 and 5,000,000 respectively. Sketch a graph to give the logarithm of the population at all times during the period, and use it to estimate the population in 1887. (*Union of Lancashire and Cheshire Institutes.*)

7. A census of the population of England and Wales was taken in 1931. Another census is to be taken in 1941. How is it possible to make estimates of the population of England and Wales for intervening years? (*Union of Lancashire and Cheshire Institutes.*)

8. The following quarterly outputs of coal are given. It is known the demand curve varies with the time of year. Calculate the correction necessary for each quarter to eliminate the seasonal movement.

	Quarters.	Output (Million Tons).		Quarters.	Output (Million Tons).
1927	1	68·3	1929	3	62·8
	2	62·6		4	67·0
	3	61·1	1930	1	70·1
	4	63·3		2	59·1
1928	1	65·4		3	56·3
	2	57·9		4	61·6
	3	56·4	1931	1	59·9
	4	61·5		2	54·8
1929	1	68·1		3	51·1
	2	62·7		4	58·0

(*Incorporated Accountants (Final).*)

9. The following table shows the month of occurrence of deaths of males from Whooping Cough and from Erysipelas in England and Wales, 1933–34. Draw a graph showing the comparatively monthly variations of mortality from these two diseases as percentage fluctuations about the average for the period.

Cause of Death.	1933.			1934.								
	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May.	June	July	Aug.	Sept.
Whooping Cough	47	36	73	73	108	130	137	118	90	47	32	28
Erysipelas	30	51	98	121	107	75	94	71	49	35	29	38

(*N.A.L.G.O.*)

10. If you were asked to make an estimate of the number of children for whom school accommodation would have to be provided under your authority in ten years' time how would you do so? (migration may be ignored). (*N.A.L.G.O.*)

11. From the following particulars as to the numbers of Insured Males unemployed in the Building and Construction Trades in the U.K., determine and plot the trend, together with a curve showing the monthly variations.

	Thousands.		Thousands.		Thousands.
1936—Jan.	417	1936—Aug.	233	1937—Mar.	280
Feb.	332	Sept.	242	Apr.	234
Mar.	270	Oct.	255	May	226
Apr.	245	Nov.	277	June	210
May	222	Dec.	308	July	222
June	221	1937—Jan.	308	Aug.	220
July	237	Feb.	290	Sept.	228

(*N.A.L.G.O.*)

CHAPTER XIX

BUSINESS BUDGETS AND BUDGETARY CONTROL

BUSINESS forecasting and budgetary control are important developments of statistical method, and both depend upon systematic planning or preparation of budgets for various business operations.

Details of any such scheme must vary considerably according to the organisation concerned, but there are fundamental principles which may be considered.

A distinction must be observed between the preparation of a budget and budgetary control. The one is the adoption of a predetermined plan of financial or other operations, and the other is the controlled execution of it by means of co-ordinated effort.

A Business Budget is necessarily a composite one, and regarding it as the master budget, the following divisions of it would normally be required.

(*a*) Sales Budget, including any selling campaign.

(*b*) Production Budget, or, in the case of a business simply engaged in trading, a budget of stock and purchases.

(*c*) Expense Budget, which will be subdivided into fixed and variable expense corresponding to the planned production and sales.

(*d*) A Plant and Tools Budget in the case of a large manufacturing concern.

(*e*) Financial Budget to cover the financing of all operations.

The Sales Budget.—As a rule the sales budget is the key to the preparation of the other budgets, but occasionally the production budget is planned first, viz.: (*a*) When the possible maximum is limited, *e.g.* certain crops; (*b*) when a certain quantity must be produced to make the units at a desired cost, *e.g.* certain engineering and foundry productions, unless produced in large quantities, cannot be made to sell at a competitive price.

An annual budget is usually prepared in the case of distributive trades and of the light industries.

A three months budget, or shorter period, is necessary where production occurs after sales orders have been received, *e.g.* heavy basic industries.

Certain types of constructional businesses may make a budget covering two to five years.

The budget preparation necessitates an analysis of market conditions, the trend, and the immediate and distant future prospects. Past performance must obviously be considered.

The budget may be: (1) by types of product, lines offered and so on, (2) by quantities and values per item, (3) by geographical sales areas or travellers' "grounds."

When values form a basis of the budget, the necessary adjustment for money values and individual prices must be made. Sometimes arithmetic averages, duly weighted for quantities anticipated, are used.

Due regard must be paid to available markets, special conditions obtaining in particular areas, the volume that the plant and organisation can handle and, not least, the labour and capital available. Reports from salesmen, agencies and special surveys may be required, and these play an important part in preparing sales budgets.

The Production Budget.—When the sales budget has been settled, the production organisation must be determined to provide the estimated requirements of each line of product, and to arrange for the raw materials, labour and finance. Production has to be planned to make the most economical use of the plant, and the cost basis is commonly adopted to correspond with the sales quotas reduced to that basis. It is only in production for stock, from which sales orders are supplied, that production budgets are really practical.

A factor of importance is the size of economical production orders, and this is ordinarily the basis of production schedules for any period, and not the estimated sales for that period, always provided that the budget limits for the budget period are observed.

Statistics should provide the necessary control over quantities produced and stocked, and the cost accounts the control over costs. Adequate stocks must be planned and controlled accordingly; excessive stocks tie up capital unnecessarily and introduce risks from deterioration and fluctuation in prices.

With regard to labour, planned production may overcome seasonal variations, and helps to ensure retention of a competent staff. Peak loads and avoidable overtime are minimised.

Manufacturing expenses can be checked against standard costs based upon the production budget. These standard costs, used by the cost accountant as measures of efficiency, are based on the projected volume of production, not on past records. Through these standard costs, and budgeted details of production, budgetary control can be effected.

The Budget of Trading Stocks carried by wholesalers and retailers, based on an estimate of annual requirements, involves also the preparation of monthly, weekly or even daily sales budgets, departmentally. Based on these estimates, buying limits are notified to the departmental buyers. Monthly, weekly or even daily statistics provide the necessary information for budgetary control, by presenting details of planned and actual figures. Often comparative figures for a previous period are included.

Expense Budget.—The budget of expenses enables the management to set up standard costs or ratios under various sectional headings, *e.g.* advertising, selling staff, delivery costs, etc. When properly controlled upon details afforded by regular statistical reports any excess beyond that warranted by the volume of sales would call for inquiry and curtailment of future excesses, except in so far as changed conditions necessitate them. Usually the percentage or unit costs under each heading of expense are calculated by the cost department, and suitable statistical reports can be prepared.

Co-ordination should be arranged between all departments, to ensure the closest realisation of all budgets as a whole. In addition to internal factors, it is often necessary to obtain much information from outside. It is the duty of the statistical department to co-operate with all departments by affording useful statistical information, and with the administrative officers by preparing such reports as will enable them to keep in close touch with data necessary for control. It is not, however, the responsibility of the department to prepare the various budgets. The budgets are seldom rigid, and being merely carefully planned estimates, may be modified when conditions necessitate this course. The real objective is general efficiency which is the foundation of profit earning.

The value of statistical reports depends largely on the method of presentation, and this in turn depends upon the ability of the executive officials to use statistics. Some can understand and prefer textual or tabular reports, others appreciate better graphs and diagrams, and the statistical department should act accordingly.

Business Forecasting is not so much the estimation of certain figures of sales, production, profits, etc., as the analysis of known data, internal and external, in a manner which will enable policy to be determined to meet probable future conditions to the best advantage.

Two aspects of business forecasting arise: (*a*) the analysis of past conditions, (*b*) the analysis of current conditions in relation to a probable future tendency.

Historical Analysis shows the course that business has followed, and, as observed in the preceding chapter, there is value in knowing the extent of seasonal variations, long-period trend, and cyclical and erratic fluctuations. The course and extent of such movements are usually more clearly seen when graphic methods are utilised. The utility of statistical analysis of variations and trend for purposes of business forecasting will be apparent. In the interpretation of historical analyses, however, care must be taken to consider any new factors affecting current conditions.

Analysis of Current Conditions will often reveal factors influencing business in a certain direction, and in many cases the effects are measurable. Examples are new inventions and new discoveries, or developments in public tastes, all of which have an important bearing when business forecasting is attempted. Such factors may alter sequential changes observed by historical analysis, and consequently any new influences should be borne in mind when arriving at conclusions or when attempting to forecast the probable course of business.

The Business Index or "Barometer" is one of the modern devices for assisting business men to follow the trend, cyclical movements and short-time oscillations of data, and if careful consideration is given to all current conditions which may influence business, it facilitates various forms of business forecasting. It may take the form of a tabulation of index numbers over a period of years, or, as is frequently the case, these numbers may be shown in the form of a graphic chart.

An index of this type may refer to general conditions of trade, production or finance, or to a particular trade or industry, or even to an individual business. One of the best known examples is the Board of Trade Index of Wholesale Prices, an index based

QUARTERLY INDEX NUMBERS OF PRODUCTION.

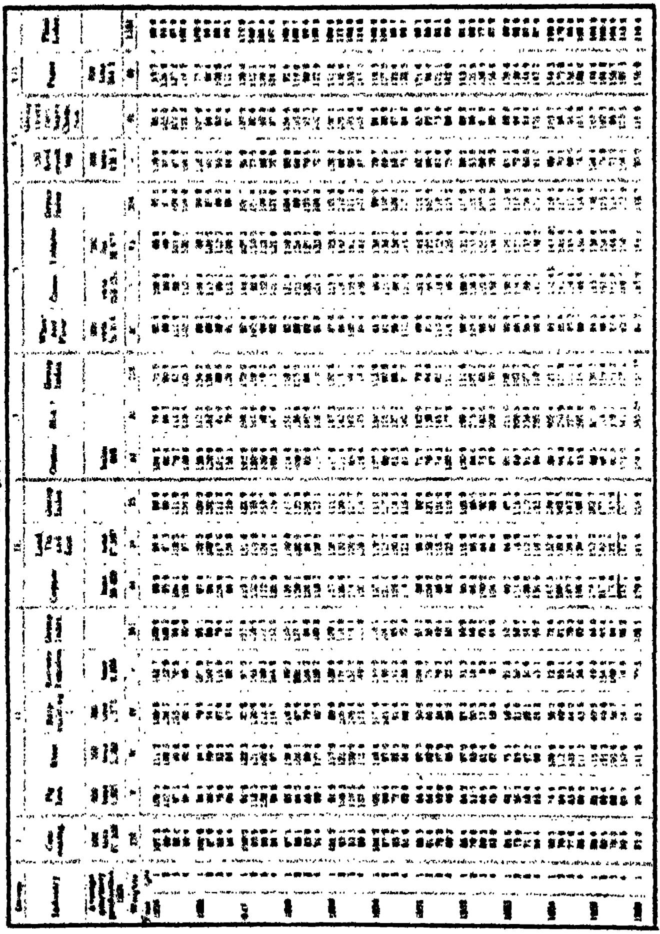

Fig. 93.—The Index Numbers in this Table are shown in Graph Form in Fig. 94.

formerly on the geometric mean of the changes in prices of 150 commodities, but since January 1935 on 200, and the base year is now 1930, formerly 1924 (see Figs. 67*a* and 67). Other notable examples are those of the *Economist* and the *Statist*, the latter being based on 45 commodities (19 foodstuffs and 26 raw materials) grouped under six headings.

Examples of tabulated index numbers are given in Figs. 93, 67*a*, and 97. Fig. 93 is a table showing "Quarterly Index Numbers of Production," Fig. 67*a* "Index Numbers of Prices and Wages," and Fig. 97 Financial Statistics relating to Bank Clearings, Money Rates and Index Numbers of Prices of Stock and Shares. The

QUARTERLY INDEX OF PRODUCTION.

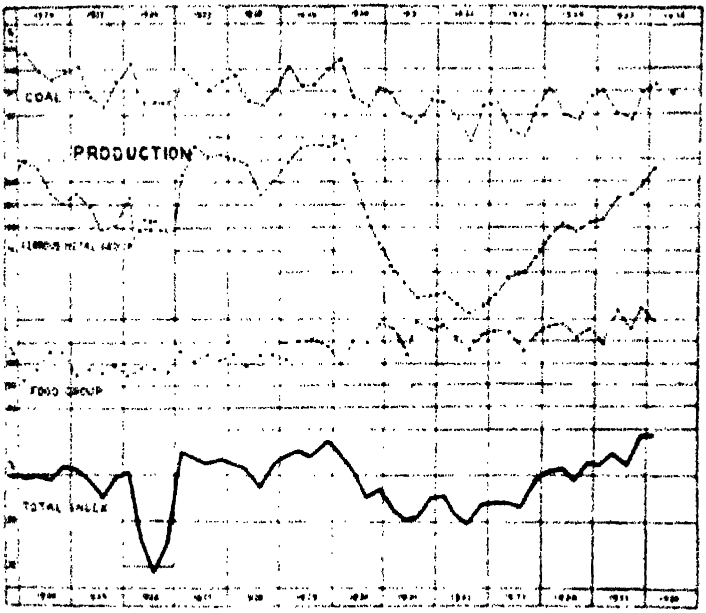

FIG. 94.—PREPARED FROM DATA GIVEN IN FIG. 93.

corresponding graphs of these, shown in Figs. 94, 67, and 80, give a readier view of the movements recorded in the tabulations. These figures are reproduced by kind permission of the London and Cambridge Economic Service, from its *Monthly Bulletin* of April 1936.

It will be noticed that in the graph (Fig. 94) the index numbers representing the physical volume of production are plotted quarterly, from which the fairly regular seasonal fluctuations in coal production are clearly brought out, and at the same time the general long-period trend can also be observed. The second curve on the graph, which refers to the ferrous metal group (consisting of pig-iron, steel, shipbuilding and railway vehicles), does not exhibit a seasonal oscillation. The trend of various price indices is seen in the graph in Fig. 67.

There are a number of different organisations issuing reports and charts relating to general business conditions, and to specific aspects of business and finance. Not all are based on purely systematic data, but also upon supplementary information obtained from bankers, manufacturers and merchants. Many banks publish statistical reports monthly, and indicate various aspects of business outlook which are valuable, as much information is available to them on actual conditions and tendencies in important industries and lines of business. Details of published statistics are given in the next chapter.

Not all businesses follow the general trend, and consequently various "barometers" dealing with important industries and particular types of product are prepared by various organisations and trade journals. Much information of this type is published from time to time by the London and Cambridge Economic Service.

A business index gives a general view of conditions that have passed, and although business forecasts based on it may often be faulty for various reasons, or may be indefinite in various respects, when prepared in the light of current conditions and trends, a fair reliance can be placed upon it, provided the necessary limitations are understood.

The Uses of Index Numbers in connection with various commercial problems both of a general nature and of individual businesses include :—

(*a*) Guidance when considering sales campaigns. Indications of rising prices and improving general prosperity may lead to a decision to embark on a special advertising scheme to increase turnover or to introduce some new article to the public at a favourable time.

(*b*) Guidance as to the advantages or dangers of making bulk purchases, or forward contracts having regard to price tendencies.

(*c*) Guidance in regard to the advisability of embarking on heavy capital investment in plant, building operations and other forms of development.

(*d*) In the case of the Cost of Living Index, this affords

> assistance in dealing with wages problems, particularly if used in conjunction with information concerning current rates of wages paid in other concerns and industries. The retention of suitable employees, and avoidance of the not inconsiderable loss caused by heavy labour turnover, may be ensured by opportune adjustment of wage rates. It is always to an employer's advantage also to have a contented personnel.

The Cost of Living Index Number, and certain other index numbers officially compiled, will be dealt with further in another chapter.

Interpretation of Statistics.—It will be apparent from the above discussion that ability to interpret statistical analyses and reports is an important feature of management, and essential to reasonable forecasting. It is the duty of the statistical department to prepare and present the facts in the most appropriate manner, but it is an equally important duty to indicate the more important conclusions deducible. This interpretation is valuable to the business executive, even though he may, and often does, observe additional interpretations or even modifications. Statistics are the key to budgetary methods, budgetary control and business forecasting.

QUESTIONS

1. What is meant by "budgetary control" in a business? Discuss the view that the two chief things to estimate are the future production cost and the future selling price of the goods. (*London Chamber of Commerce.*)

2. Describe sources of statistical information on the conditions of any industry known to you. (*London Chamber of Commerce.*)

3. Explain what is meant by a "moving average." The following index number of wholesale prices is quoted from 1900 to 1929. Show how to obtain a smooth curve indicating the general movement of prices.

1900	75	1910	78	1920	251
1901	70	1911	80	1921	155
1902	69	1912	85	1922	131
1903	69	1913	85	1923	129
1904	70	1914	85	1924	139
1905	72	1915	108	1925	136
1906	77	1916	136	1926	126
1907	80	1917	175	1927	122
1908	73	1918	192	1928	120
1909	74	1919	206	1929	115

(*London Chamber of Commerce.*)

4. State the different statistical methods by which phenomena at different dates may be compared. (*Incorporated Accountants* (*Final*).)

5. Given the monthly statistics for the ten past years of the sales quantities of an article with an inelastic demand, how would you form an opinion of the results of a current year as the separate monthly figures for that year became gradually available? (*Incorporated Accountants* (*Final*).)

CHAPTER XX

PUBLISHED STATISTICS

Their Interpretation and Value

It has been mentioned previously that much useful statistical information for business purposes may be obtained from external sources, and in particular from published statistics. In using such statistics it is necessary to understand what they comprise in each case, so that proper interpretation of them may be made before applying them for business purposes. Generally speaking, it is probable that many business men do not realise the value of published statistical reports, and fail to take the fullest advantage of them, notwithstanding the valuable information which can be utilised in various ways. Some of the published statistics will now be briefly considered. References to others may be found in the Annual Guide to Current Official Statistics published by H.M. Stationery Office.

I. Census Reports

The Census of Population.—This has been taken every ten years since 1801, but is now to be taken every five years. The United States Census is also decennial.

The forms or questionnaires required to be filled in for every dwelling afford, when completed, information relating to every person at the particular date named, viz.: names, relationship to the householder, age, sex, civil condition (married, single, etc.), birthplace and nationality, number of children, personal occupation, industry in which employed, and particulars of persons over 14 not following an occupation for profit.

The Value of the Census for governmental purposes, such as local government, old age pensions, educational facilities, housing, health matters, etc., is great, but it is also important for the manufacturer and trade distributor, inasmuch as considerable information is given about consumers, their distribution and occupations, the utility of which will now be summarised.

1. *Density of Population by Areas.*—As the Census Report

shows the distribution of people, the density of population in various areas can be ascertained. When using totals for various towns the figures relating to suburbs and adjacent places may have to be extracted and included to obtain effective totals. From this quantitative markets can be estimated by producers and trade distributors, wholesale and retail, for the purpose of their marketing organisation.

The large distributor can plan for a suitable number of salesmen for densely and sparsely populated areas, and the disposition of area depots.

Sales quotas can be allocated either on a general per capita calculation, or, as may be necessary for some classes of goods (*e.g.* ladies' and children's wear) on the basis of the number of females, of children, or of age. The statistics afford the requisite information in the various lists prepared.

2. *The Type of Consumer.*—The occupational distribution lists of the people, by professions, trades, etc., over towns and areas, will give some guidance as to the probable purchasing power. These statistics combined with Ministry of Labour statistics of wages are particularly useful for localised advertising and selling compaigns, and for planning distribution according to likely demand for commodities in various areas. It is a great help to know number and "quality" of area population when estimating potential demand. Family budget investigations may serve in a similar manner.

3. *Availability of Labour* is disclosed by the occupational lists, which may be of value to the producer when deciding upon the locality for his factory site. The labour supply both skilled and unskilled is as important as proximity to supplies of raw materials and transport facilities. The question of future labour supply is covered by the age statistics, and the numbers of male and female workers probably available is ascertainable. Not less important is the number of employers in an area likely to be competing for labour, a matter which may affect wage rates if the labour supply is limited.

The Strength of Markets.—The large producer may utilise the occupational lists to find the number of traders (*e.g.* retailers and wholesalers), in his product in various districts, this being information which will assist him in organising his sales campaign. Specialised advertising suitable to the general type of industry and labour in various areas may be arranged from the same

information. Tendency to decline or increase may be revealed in some areas, and the extent of change, if marked, should be allowed for when reviewing sales efforts. A rapidly growing town should show increasing purchases of a large distributor's goods.

II. PRICE INDEX NUMBERS

Day-to-day prices of raw materials and manufactured products are readily available from various sources, but for the systematic study of the trend of prices index numbers must be used.

Index Numbers are calculated in several ways, the chief of which have been demonstrated in Chapter XV. Some are based on geometric averages, others on arithmetic averages; there are several methods of weighting; and different bases are used.

The chief features of the Board of Trade Index Number of Wholesale Prices are dealt with below as supplementary to the general remarks given in the previous chapter.

Board of Trade Index Number of Wholesale Prices.—The price relatives are calculated on the *Chain Base Method*. The aggregate index number is obtained by taking the *Geometric Average* of changes in prices of 200 commodities. The base year is 1930, but formerly was 1924. The advantage of the geometric average is that it does not give undue weight to extreme prices. Average import and export prices are used and the data relate to average prices for the month preceding. In some cases two or more quotations are averaged to obtain the price of a commodity.

The groups of items included are :—

(i) *Cereals.*—Wheat, flour, bread, barley, oats, oatmeal, maize, rice and tapioca.

(ii) *Meat and Fish.*—Beef, mutton, pork, bacon, eggs and fish.

(iii) *Other Foods.*—Milk, butter, coconut oil, cheese, potatoes, onions, apples, oranges, sugar, tea, coffee, cocoa and tobacco.

(iv) *Iron and Steel.*—Pig iron, wrought iron, steel, and rolled, forged and similar steel products.

(v) *Coal.*—Ten items of export, household and navigation coal.

(vi) *Other Metals and Minerals.*—Petroleum, copper, lead, tin, zinc, and nickel.

(vii) *Cotton.*—American and Egyptian raw cotton and yarns, cloths.

(viii) *Wool.*—Raw wool and yarns, wool rags, tops in oil.

(ix) *Other Textiles.*—Linen yarns, raw silk and silk tissues, raw jute and hemp.

(x) *Chemicals and Oils.*—Chemicals, tallow, linseed oil.

(xi) *Miscellaneous Materials.*—Paper, leather, rubber, building materials, timber, bricks, stone and glass.

A reclassification of the non-food groups is given under the following headings :—

Industrial Products (excluding fuel), Basic Materials, Intermediate Products, Manufactured Articles and, separately, Building Materials.

The weighting is effected by taking additional quotations in the groups proportionate to the importance of each as determined by the 1930 Census of Production, ordinary weighting not being used.

The chain base method, each price calculated as a percentage of the figure of the previous year, has two advantages : (*a*) current commercial needs are served by making comparisons possible with recent conditions, (*b*) substitutions in commodities or grades can be made when one is no longer in demand, without disturbing the sequence of numbers. This is possible because there is no dependence on a fixed base year.

Besides the aggregate index number, one is calculated for the first three groups (foods) and one for the remainder (non-food) groups. The Index is published monthly in the Board of Trade Journal.

The disadvantage of such an index number is, that although it shows the trend of prices, it is nothing more than an approximation which is not conclusive. Commodities included are not necessarily equally available, consumed or demanded, nor is any separate significance shown for quality and grade.

Other Price Index Numbers.

"*Economist*" *Index.* Published monthly, based on the unweighted geometric average of changes in prices of 58 items, and shown as a percentage of 1913.

A fortnightly index of sterling and international prices, as a percentage of September 25th, 1931, is also published.

"*Statist*" (*Sauerbeck*) *Index*, published monthly, is an unweighted average of percentage changes in 45 items, the base being the averages of 1867–1877. These index numbers are shown in Fig. 67*a*, but it should be noticed that they have been proportionately transferred to percentages of the 1924 average by the London and Cambridge Economic Service, by whose permission the table has been reproduced.

"*Financial Times*" *Index*, published monthly and weekly, is based on 75 commodities. For the monthly index the previous year is the base; for the weekly index it is September 1931.

"*Times*" *Index*, published on the first of each month since 1920, is the arithmetic average of changes in prices of about 100 items as a percentage of the average for 1913.

Reuter's Index of wholesale prices daily is the weighted geometric average of prices of 21 primary commodities including metals. As it is a world price index the weighting is proportional to the importance of the groups in international trade, viz.; weights in brackets, wheat (17), rice (6), maize (5), sugar (9), cocoa (2), copra (2), pepper (1); cotton (14), jute (2), hemp (1); gold (8), silver (4), copper (6), tin (5), lead (2), spelter (1), linseed (4), ground nuts (3), soya (2); rubber (5), shellac (1). Base September 18th, 1931; published since 1933.

Ministry of Agriculture's Index of prices of agricultural produce, published monthly in the *Agricultural Market Report*, is the weighted arithmetic average of the percentage price changes, shown as percentages of the corresponding month 1911–1913. The weights are proportionate to the values determined by the Agricultural Census of Production, 1908.

Ministry of Labour's Retail Food Index is published monthly in the *Labour Gazette* as a subsidiary to the Cost of Living Index described below.

The method of constructing an index number has already been described in Chapter XV, and the official and other index numbers are shown in Fig. 67*a*, and in the graph in Fig. 67.

Cost of Living Index Number.—Unlike the index number just described, the Cost of Living Index Number is based on retail prices. These prices have little relation to the wholesale prices, for retail prices vary much both between one part of the country and another, and in individual localities. Different considerations, therefore, must enter into its compilation.

The Ministry of Labour Cost of Living Index Number published on the 1st of each month is intended to measure the average cost of maintaining unchanged the pre-war standard of living of the working classes for the specific purpose of ascertaining what increase has occurred. It is in the form of percentage *increases* over July 1914. The index numbers for the years 1931 to 193 are shown in Fig. 67*a*, and in the graph in Fig. 67, but these figure are given as a *percentage* of the average 1924 level instead of onl the increase.

No allowance is made for any change in the standard of livin since before the War, and no question of the adequacy of tha standard is considered.

The cost of living varied as between families in pre-war years and that obtains to-day; also the allocation of expenditure o different items necessarily differs as between one person an another. Consequently the increase in cost of living is expresse as a percentage; for such a purpose it is clear that the faires practical calculation can be only a reasonable general averag Factors that operate against the justness of the Cost of Livin Index Number as a true basis of comparison are: (*a*) Varyin prices and unequal rates of change of individual commoditie used have caused variations in the amount spent on each (*b*) changed earnings have resulted in a higher standard of livin in very many cases.

The Cost of Living Index has been used largely in connectio with wages negotiations. Since 1914 wages have been increase in many instances on the basis of the Cost of Living Inde Number; in several industries changes in wages paid take plac automatically according to a sliding scale governed by the Inde Number; and in many instances in recent years it has bee accepted as a basis for deciding reductions.

The Objections to the Cost of Living Index Number may b summarised as follows :—

(*a*) The 1914 base is out of date. It is claimed that a ne comprehensive analysis of working-class household budget should be made, as present day distribution of wage expenditure is considerably changed, and the "weighting" of items needs revision. (A special enquiry to collec new data for revision of the weighting and content of th index number has been announced.)

(*b*) It fails to allow for definite changes in the standard o living.

(*c*) Its use for the adjustment of wages to correspond wit purchasing power of money is felt only by the lowe grades of employees, and that the higher paid employee suffer a diminution of real wages because the index is not representative of their standard of living.

(*d*) It fails to include certain luxuries which are now widely enjoyed almost as necessaries.

(*e*) The method of compilation overstates the percentage increase owing to the fact that as certain commodities become dearer less of them is bought, and more is spent on relatively cheaper things.

(*f*) Allowance should be made for the benefits of public social services which have considerably increased since 1914.

The Uses of the Cost of Living Index are chiefly in connection with wages adjustments. It serves as one form of measuring the purchasing power of money. Other aspects are referred to on another page.

The Compilation of the Cost of Living Index Number.—A weighted arithmetic average is used and the items included are :—

Food, viz.: beef, mutton, bacon, fish, flour, bread, potatoes, tea, sugar, milk, butter, margarine, cheese and eggs. Fruit and vegetables are omitted, as they vary so greatly both as to prices and supplies.

Prices are obtained from over 600 towns and villages, and reports of prices used are obtained from over 5000 retailers. Weighted averages are used, the weights being based on average expenditure ascertained from 1944 working-class budgets by the Board of Trade in 1904. Negligible change occurred between 1904 and 1914, the base year used.

Rent.—Details are supplied by town clerks of the principal towns, supplemented by other reports, and include both controlled and de controlled rents.

Clothing.—Includes : men's suits and overcoats, woollen and cotton materials and garments (outer and under), hosiery and boots, all chiefly relatively low-priced grades. Prices are obtained from about 300 retailers in about 80 towns.

Price relatives for each quotation are worked on the chain-base method, and then a weighted average is taken.

Fuel and Light, viz.: coal, gas, oil, candles and matches. Prices are obtained from 25 to 30 towns, an average of each is taken by totalling the quotations and dividing by the number of items. A fixed, not a chain, base is used. The weights used are coal 6, gas 3, other items combined 1.

Other Items, viz.: soap, soda, ironmongery, brushes and pottery, tobacco and cigarettes, fares, newspapers. The prices are obtained from retailers and public announcements. They do not affect the general figure very greatly. The percentages of increase are averaged and worked into a weighted average on the fixed base principle.

A Combined Weighted Average of the five groups is then calculated, the weights used being: Food, 7½; Rent and Rates, 2; Clothing, 1½; Fuel and Light, 1; Other Items, ½, *i.e.* the percentage increase for each group is multiplied by the stated weight, the products totalled and divided by 12½, the sum of the weights. The omitted items are so small that they would not affect the general average appreciably (see p. 104).

III. WAGE STATISTICS

The principal statistics of wages are those published monthly by the Ministry of Labour in the *Labour Gazette*, and annually in the *Abstract of Labour Statistics for the United Kingdom*. Below are the more important matters covered:—

(1) Principal changes in rates of wages published month by month. This table gives detailed particulars of increases and decreases in rates of pay in each industry and district in the United Kingdom; the date from which the change took effect and the classes of workpeople affected.

(2) The above data are used to present an annual summary showing for a series of years: (*a*) The number of individuals who have been affected by wage changes, separating those who have had increases and decreases respectively. (*b*) The estimated net weekly amount of change (increase and decrease separately) caused by the alterations in rates paid.

(3) The weekly amounts of change in wages due to rate changes but analysed according to the method under which the changes took place, viz.: (*a*) By Mediation or Arbitration; (*b*) under Sliding Scales governed by (i) Cost of Living, (ii) Selling Prices or Proceeds of Industry; (*c*) by Joint Standing Committees; (*d*) by Direct Negotiation. The table is prepared in sections for different groups of industry.

(4) A comprehensive table showing the relative level of wages for adult workers in a large number of industries, and in many cases giving this information for each grade of occupation separately.

The Ministry of Labour Index is based upon changes in rates of wages in 32 industries using a weighted average for each date, expressed as a percentage of the average for 1924. The Index represents the average percentage increase in weekly full-time wages generally compared with July 1914.

In addition to the above, reports on general inquiries into wages and earnings have been given, showing, for a very comprehensive list of industries, the average weekly earnings of workpeople. Separate lists deal with male and female workers' earnings. The reports are based on returns made voluntarily, but are regarded as reasonably representative of average earnings.

IV. Statistics of Trade

The External Trade of the United Kingdom.—Statistics are published officially by the Board of Trade. They are issued in considerable detail monthly, and summarised in a bulky publication entitled *Annual Statement of the Trade of the United Kingdom*, in four volumes. The information is as follows :—

(*a*) Commodities imported and re-exported.
(*b*) Commodities classified by countries of consignment.
(*c*) Exports classified by countries.
(*d*) Trade with each country, and the exports and imports at each port.

Quantities and values are given. In connection with the latter it should be observed that exports are based on free on board (f.o.b.) values ; imports values include cost, insurance and freight (*i.e.* c.i.f. values).

The Statistical Abstract for the United Kingdom, issued annually in August, gives exhaustive statistics of trade and shipping, and detailed commodity reports, and should be consulted by all students as a guide to methods of constructing tabulations.

The primary data for these statistics are obtained : (*a*) *For Exports*, from the statement of quantity and value compulsorily supplied by exporters to the customs officers. Ships are permitted to leave port only when all cargo on board has been so accounted for or "cleared." (b) *For Imports*, from forms completed before goods are passed through the customs, *i.e.* before they are handled or are assessed for duty. *If prices include delivery charges, the value is adjusted for the trade statistics.*

The following facts should be noted :—

Goods landed "in bond" and transhipped at the same, or another port, are excluded from above totals, but are included in the total of "general" imports and exports.

Goods landed, which are eventually shipped again, are declared as of foreign or colonial origin, and fall into a separate total of "goods for re-exportation," and their values are usually greater by the costs of landing and handling in this country.

Goods for use or consumption in the United Kingdom, and exported goods which have been produced or in any way processed in the country are scheduled under "*Special Imports and Exports.*"

Statistics of commodities are classified under the following headings for both imports and exports :—

(i) Food, drink and tobacco.

(ii) Raw materials and materials mainly unmanufactured. (Articles which are the finished product of one process or industry and become the raw material of another process of manufacture, *e.g.* pig iron, yarn, refined oils, etc., are *not* included here, but are classified in the group 3.)

(iii) Articles wholly or mainly manufactured.

(iv) Unclassified.

Index Numbers of Volume and Average Value of External Trade are published quarterly in the *Board of Trade Journal.* The base year is 1930.

Balance of Payments of the United Kingdom.—The Board of Trade gives an annual estimate of the country's balance of trade, taking into account "invisible" exports and imports, *i.e.* those items which are not exports and imports of tangible goods dealt with in the trade statistics already described.

Examples of *Invisible Exports* are : (*a*) shipping services, (*b*) investments and loans abroad, (*c*) commissions earned from persons abroad (on acceptance credits, bill discounting, insurance, brokering, etc.) and (*d*) Government services and loans abroad for which payment is made to the Government.

Examples of *Invisible Imports* are similar to those given for invisible exports, but in the reverse direction, *e.g.* interest payable abroad on loans received from other countries, services rendered by persons abroad.

There is an Annual Statement in the *Board of Trade Journal* of (*a*) the Visible Trade Balance, (*b*) the Balance of Payments, which includes the invisible exports and imports referred to above.

The title is "Balance of Credits and Debits in the Transactions (other than the lending and repayment of capital) between the United Kingdom and all Other Countries."

Statistics of Internal Trade.—No comprehensive continuous statistics are published, but a monthly Statement of Retail Trade showing percentage comparisons of sales at selling values and stocks at cost with the same month of the preceding year, are published in the *Board of Trade Journal* and the Bank of England's *Monthly Statistical Summary*. They are prepared from statistics of certain multiple retailers, departmental stores and co-operative stores. This index shows the change in aggregate value of retail sales for Great Britain as a whole, and for five geographical divisions.

V. Finance

The leading banks publish financial statistics in their Monthly Reviews. Monthly returns of "Clearings" by the London Clearing Banks are published, as are cheque clearings in London and provincial centres.

The Bank of England Return appears every Thursday in the daily papers.

Various index numbers relating to Stock Exchange price levels are published, e.g. *Bankers' Magazine Index* calculated from total market values of about 390 securities; base December 1921; *Actuaries' Index Numbers*, the geometric mean of price ratios of 319 securities expressed as a percentage of December 1928. Middle market prices in London Official Lists are used; the *Financial News* publishes a daily index; the *Financial Times*, a weekly and monthly index; the *Investors' Chronicle*, a monthly index based on 176 quotations in five groups.

Valuable statistics relating to Bank Clearings, Treasury Bills, Money Rates, New Capital Issues, and index numbers of prices of stocks and shares are published by the London and Cambridge Economic Service, in the form shown in Figs. 86 and 97, reproduced by kind permission from their *Monthly Bulletin* issued in April 1936.

VI. Production

The Statistical Abstract contains statistics as to the amount of land in use, and the yield year by year of various crops, based on data collected by the Ministry of Agriculture and Fisheries. Annual *Agricultural Statistics* are published separately. *The*

FINANCE

[illegible]

* NORMAL SEASONAL CHANGE REMOVED.

STOCKS & SHARES— Index Nos. of Prices and Yield as percentage of 1924 level, on 15th of month. Sensitive Index.—Geometric Mean of monthly percentage changes.

NEW CAPITAL ISSUES— Issues during month in Gt. Britain, (a) for U.K., (b) for Abroad, excluding Government loans, etc.—As published by the MIDLAND BANK, LTD.

BANK CLEARINGS— Total of Town Clearings (i.e. excluding Metropolitan) of London Bankers' Clearing House for 4 weeks covering 2 Stock Exchange settlement days. Canada settle- [illegible] of following month. Country Clearings of London Bankers' [illegible] Provincial Clearings for 11 towns [illegible] totals for 24 [illegible]

[illegible] Issues [illegible]

Reports of the Mines Department gives statistics of the quantity and value of output of coal, copper, lead, tin, zinc and other metals. These are also summarised in the Statistical Abstract. The Textile Trades statistics are published in various trade journals. Wool statistics are published by the Bradford Chamber of Commerce. Annual Reports and publicity statistics of various Public Authorities (Water, Electricity and Gas) contain useful statistics, as do the Reports of the various Marketing Boards, *e.g.* Potatoes, Pig and Milk Boards.

The Census of Production.—The official census conducted by the Board of Trade is limited to manufacturing industries, mining and quarrying, building, productive work of public utility undertakings, local authorities and Government Departments.

Censuses were taken in 1907, 1912, 1924 and 1930.

The following explanatory observations, chiefly relating to the 1930 Census, should be noted :—

(*a*) Small businesses employing not more than ten persons were excluded.

(*b*) Goods sold in the same condition as received were excluded.

(*c*) The value of the output represents net selling value as packed ready for sale of goods delivered in the census year, plus stock on hand at the end of the year, less stock at the beginning of the year. Excluded from the amounts invoiced were: discounts, carriage outwards, packing cases returnable. For work done " on commission " or " for the trade," only the price for doing the work was included as " output."

(*d*) Materials used included materials purchased for use in production, including all fuel, gas, electricity, and oil, packing and workshop materials, and materials used for repairs to plant and buildings.

(*e*) Net output consisted of gross output, less total cost of materials used and of work done for the producer by outside firms. It represents the total out of which all wages, expenses and profits have to be provided. The net output total represents therefore the value added to materials by manufacture, thereby preventing duplication of values which would have occurred had gross output been aggregated.

The Results of the Census are published in a series of

Preliminary Reports. Examples of the type of statistics published are:—

Principal Products of Industries, showing quantities and values, average selling prices, amounts exported, imported and held for consumption in the United Kingdom.

Cost of Materials Used, and the value of the Net Output (as defined above) by industries; also the value of net annual output per person employed.

Employed Persons in each industry, tabulated to show separately: (*a*) operatives, (*b*) administrative, technical and clerical staff, in each case classified as to males and females, under eighteen, and all ages.

Mechanical Power is tabulated to show the total horse-power of prime movers and electricity (generated and purchased separately), and the total kilowatts provided by electric generators.

A Summarised Tabulation of all the above detailed statistics for each industry is prepared, giving the comparative figures for the previous census year.

Indices of Production.

The Board of Trade Index of Industrial Production.—This index relates to the net output of the principal industries, "net output" being the excess of the gross value of the production over the cost value of materials used in their manufacture. The year 1930 is now used as the base, instead of 1924.

Weights are used—namely, a weight proportionate formerly to the net output ascertained for 1924, but (since 1934) that for 1930.

Information for the compilation is obtained from official returns, together with data as to production and wages paid obtained from various trade associations and federations, trade papers and individual firms.

The London and Cambridge Economic Service Index of Physical Volume of Production.—The annual index is on the base 1924 = 100, a separate index being compiled for the following groups:—

(1) Agriculture (excluded in the Board of Trade Index); (2) Minerals; (3) Iron, Steel, Engineering and Shipbuilding; (4) Non-ferrous Metal Trades; (5) Textiles; (6) Food, Drink and Tobacco; (7) Chemical and Allied Trades; (8) Paper, Printing, etc.; (9) Leather Trade; (10) Rubber Trade; (11) Building and Contracting.

The weights used are numbers proportional to the net output as indicated by the 1924 Census of Production.

A combined index number covering all groups is included. A quarterly index on the same lines is also calculated. (See Figs. 93 and 94.)

International Indices of Production have been prepared by the League of Nations Economic Intelligence Service and published by that body in Memoranda of Production and Trade. These, and other statistics relating to industry which the League prepares, will no doubt become of increasing importance owing to the facilities afforded to the League by many countries.

VII. TRANSPORT

Shipping.

The Annual Statement of Navigation and Shipping, published by the Board of Trade, gives detailed statistics for the United Kingdom. The Statistical Abstract summarises all these statistics.

Shipping *Entered*.—A ship's papers must be produced showing: (*a*) from where she has come; (*b*) where she last broke bulk; (*c*) the cargo carried; (*d*) the ship's tonnage. Every ship entering a port is recorded as "arrived" (excepting when entering for shelter).

Shipping *Cleared*.—A ship departing must "clear" from the port by producing the ship's papers, showing destination and tonnage, and details as to quantity and value of all cargo have been declared.

Separate statistics of steam- and sailing-ships are prepared; also of foreign and coastwise.

Tonnage in British shipping statistics means the registered net tonnage. This is the gross tonnage (viz. the number of "100 cubic feet" contained in a ship) less the space occupied by engines, bunkers, quarters, etc. (*i.e.* non-cargo carrying space). Registered net tonnage is thus the ton-measurement cargo carrying capacity of the ship. A ship ton is 100 cubic feet irrespective of weight.

It should be noticed that displacement tonnage and deadweight (or burden) tonnage do not enter into official statistics.

The Chamber of Shipping Index, published in the *Statist* monthly, is the geometric average of freight changes on 21 routes, given as a percentage of 1920.

The Economist Freight Index is the arithmetic average of six regional groups of shipping freights based on 28 items, and is given as a percentage of 1898–1913 average.

Useful statistics are published by the Port of London Authority, and the Mersey Docks and Harbour Boards.

Railway Statistics.—The Ministry of Transport issues annually *Railway Statistics*; *Railway Companies (Staff)*, together with a monthly report on traffic earnings, operation, and the quantity of selected classes of goods carried. Excellent statistical reports are published, including graphs, by the London Passenger Transport Board.

Ton-miles which are recorded are obtained by multiplying the number of tons in each consignment by the number of miles each is carried. Wagon-miles equals the number of wagons × number of miles run. Train-miles is the aggregate miles run; engine-hours is the aggregate running hours excluding shunting.

VIII. EMPLOYMENT AND UNEMPLOYMENT

The Ministry of Labour Gazette publishes monthly statistics of employment and unemployment, based on national unemployment insurance data and employers' returns, voluntarily made.

The statistics of workpeople employed do not comprise *all* employed, but only those included in the returns received.

Previous records issued from 1893 to 1927 in the *Labour Gazette* were based on returns by Trade Unions.

The Unemployment Statistics are chiefly based on the returns from Employment Exchanges operated under the Unemployment Insurance Acts. The numbers include insured persons so registering at the Employment Exchanges, plus any whose "Unemployment books" have remained lodged, but of whom no information as to whereabouts is known (these are only so counted for a period of two months). Any unemployed disqualified by trade dispute regulations and sickness are excluded. Also excluded (since January 1928) are persons aged 65 and over, these being no longer insurable persons.

Monthly figures are also published by the Ministry of Labour, showing the total number of *applicants for employment* registered by the Employment Exchanges.

Index numbers of (*a*) insured persons, (*b*) employed insured persons in each of 100 industries as a percentage of 1923 are published annually.

In reading the two sets of statistics referred to above, the distinction must be noticed as to the meaning of "unemployed person." In the first set only insured persons are included; in the second others may be included. For the form in which the detailed statistics are published the *Ministry of Labour Gazette* should be inspected.

IX. OVERSEAS STATISTICS

Data relating to Foreign Countries.—Official Statistics are published by all the principal countries.

Much valuable data and statistics, including many of a comparative nature between countries, are published by the League of Nations and the International Labour Office. A *Monthly Bulletin of Statistics* and the *Statistical Year Book* is issued by the League.

The *Statistical Year Book* gives statistics concerning labour conditions, movement of population, international trade, production, prices and public finance.

The *Monthly Bulletin of Statistics* contains tables, graphs and comments; and the statistics, collated from indicated sources, deal with imports and exports, production, freight, prices and exchange rates, unemployment.

The publications of the International Labour Office are numerous, but the *Year Book*, and the *International Labour Review*, which deals with employment and unemployment, wages movements, retail prices and the cost of living, must be specially mentioned.

A *Statistical Summary of the Mineral Industry of the British Empire and Foreign Countries* is published annually by the Imperial Institute.

Comparable Wholesale Price Index Numbers for Eleven Countries are published quarterly by the London and Cambridge Economic Service, based on prices obtained from the International Institute of Statistics (The Hague) and other sources. The prices used are those of 16 foodstuffs and 19 raw materials.

QUESTIONS

1. Give an account of the official method of measuring changes in the retail prices of working class "necessaries of Life." (*London Chamber of Commerce.*)

2. Explain what is meant by the following terms: (*a*) Unemployed person. (*b*) Imports. (*c*) Raw Materials and Articles mainly unmanufactured. (*d*) Shipping entered and cleared with cargoes. (*London Chamber of Commerce.*)

3. Discuss the importance of exact definition in statistics. Illustrate your answer by reference to either: (*a*) Occupational Statistics, or (*b*) Statistics of the Balance of Trade. (*London Chamber of Commerce.*)

4. Show how the *per capita* consumption of commodities can be used to help in sales investigations. (*London Chamber of Commerce.*)

5. Explain and illustrate the meaning of index numbers. Show the convenience which may result from their use. What special problems are presented by the determination and employment of Index Numbers of the Cost of Living? (*London Chamber of Commerce.*)

6. What are the chief sources of information regarding the quantity and value of British imports and exports? What difficulties are encountered in the interpretation of such data? (*London Chamber of Commerce.*)

7. What help in sales investigations would you expect to obtain: (*a*) from the Census Returns? (*b*) from Family Budget Investigations? (*London Chamber of Commerce.*)

8. Give some account of the British official statistics relating to transport. (*London Chamber of Commerce.*)

9. Sketch the lines on which you would proceed to prepare an Index Number of the Cost of Living. (*Incorporated Accountants.*)

10. How would you set about constructing an Index of General Business Conditions in Britain?

Explain what you would include as significant, where you would get your data, and what use could be made of the Index when constructed. (*Corporation of Accountants.*)

11. What official statistics exist in Britain for the measurement of the results of the productive activity of the nation from year to year? What uses have they for the business community? (*Corporation of Accountants.*)

12. Give as exact a definition as possible of the term "cost of living." How far can the change in the cost of living be measured over a period in which there have been considerable modifications of diet or other changes in consumption of necessary commodities? (*Corporation of Accountants.*)

13. Give the history and the methods of the British Census of Production. What use can be made of such results by (*a*) the Chancellor of the Exchequer, and (*b*) the industrialist? (*Corporation of Accountants.*)

14. What statistical data would you require and where would you get them in order to describe comparative costs between two competing countries? (*Corporation of Accountants.*)

15. Why is the geometric mean used in the construction of index numbers of wholesale prices? (*Union of Lancashire and Cheshire Institutes.*)

16. Give a concise account of the construction of the Cost of Living Index at present in use. What criticisms can be made of this Index as a measure of changes in the post-War condition of the working-classes? To what extent, in your opinion, are these deficiencies likely to be removed in the construction of the new Index for which data are now being collected? (*N.A.L.G.O.*)

17. Describe in some detail one, and only one, of the following:—(i) the Board of Trade index-number of wholesale prices; (ii) the Earnings and Hours Enquiries of the Ministry of Labour; (iii) the statistics of unemployment published in the *Ministry of Labour Gazette*. (*N.A.L.G.O.*)

18. Write a brief account of two (and only two) of the following: (i) the Census of Production; (ii) the Census of Population; (iii) the Board of Trade Index Number of Wholesale Prices; (iv) the Ministry of Labour Index Number of the Cost of Living; (v) the Percentage of Insured Persons Unemployed. (*N.A.L.G.O.*)

19. What are index-numbers and for what purposes are they used? Give your answers concisely and illustrate your statements by examples of index-numbers in actual use. (*N.A.L.G.O.*)

CHAPTER XXI

FURTHER TYPES OF GRAPHS AND FORMS

THE method of constructing various graphic charts has already been dealt with in an earlier chapter, and now several types not described there will be considered, together with some suggested forms of useful business tabular statements.

Frequency Polygon.—Groups of data were shown in Fig. 56 in the form of a Histogram or staircase chart, in which the areas of rectangles drawn are proportional to the data recorded.

The same data may be drawn as a Frequency Polygon in which the idea of area is ignored. Points are plotted over the centres of the group intervals at distances proportional to the frequencies and then the points are connected by straight lines.

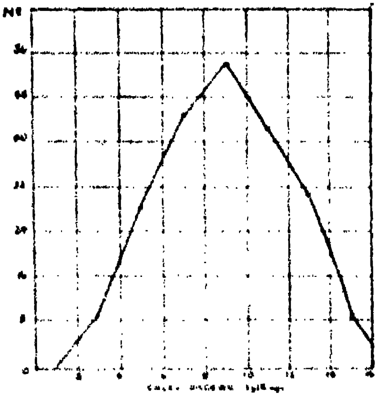

FIG. 98.—A FREQUENCY POLYGON SHOWING THE FREQUENCY DISTRIBUTION OF SIZES OF ORDERS TAKEN BY SALES DEPARTMENT.

Thus, using the same data as for Fig. 56 a Frequency Polygon may be constructed as shown in Fig. 98.

The Frequency Polygon is useful for comparative purposes, as two or more can be plotted on the same chart. Unlike the

Histogram, it cannot be used for data tabulated by unequal intervals, and it is also more difficult to smooth. In smoothing care must be taken to ensure that the total area of the curve is equal to that of the plotted polygon.

Ogives or Cumulative Frequency Charts.—The numbers of orders shown in the data for Fig. 56 may be totalled cumulatively. The totals would be 9, 36, 81, 135, 177, 207, 216 and may be read thus:—

Orders under 4s. totalled 9
" " 6s. " 36
" " 8s. " 81 and so on.

These totals are plotted in Fig. 99 and the dotted line shows how the curve may be smoothed.

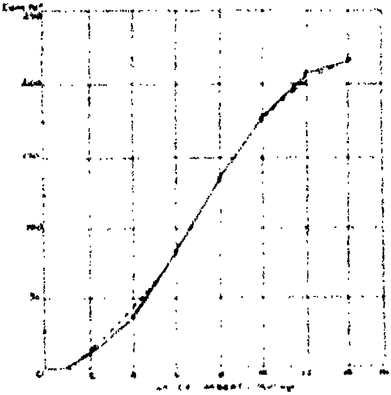

Fig. 99.—Ogive or Cumulative Frequency Chart Based on the Same Data as That Used for the Frequency Polygon in Fig. 98.

It will be apparent no difficulty is caused by inequalities in the group intervals and the ogive curve moves more regularly than that of the polygon curve.

Comparative Frequency Distributions are best shown by plotting the data on a percentage or per mille basis. Either the Frequency Polygon or the Ogive form may be used. See Figs. 101, 102 and 103 based on data in Fig. 100.

Probability Charts.—Many facts in business follow a curve of frequency which is symmetrical in its direction. The curve plotted on ordinary ordinate paper takes the bell-shape. Now, it is possible to obtain specially ruled Probability Paper, and if the same data are plotted on this, the result is a straight line. Only a small number of points need be plotted, and for the purpose of

Output of Piece Workers in Factories A and B by Quantities per Worker.

Week ending March 31st, 19....

Output Units.	Factory A.		Factory B.	
	Number.	Per cent.	Number.	Per cent.
Under 100	8	2·0	0	0
" 105	14	3·5	0	0
" 110	34	8·5	12	6·0
" 115	70	17·5	26	13·0
" 120	110	27·5	75	37·5
" 125	76	19·0	44	22·0
" 130	58	14·5	31	15·5
" 135	20	5·0	8	4·0
" 140	10	2·5	4	2·0
Total	400	100	200	100

Fig. 100.—Data for Figs. 101 to 103.

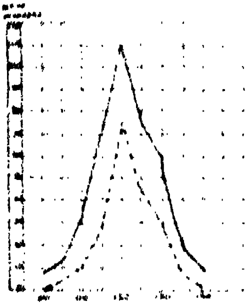

Fig. 101.—Actual Data for Two Factories.

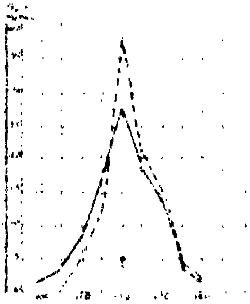

Fig. 102.—Non-cumulative Percentages.

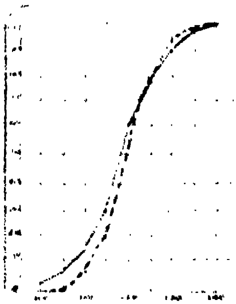

Fig. 103.—Cumulati Percentages.

forecasting probable results beyond recorded figures this line ca be extended with the great probability that the extension wil correspond with the facts to be ascertained later.

There are many uses to which this can be put, of which th following are examples :—

(1) The probable percentage of various sizes of credi accounts or orders will become bad debts.

(2) Probable number of buyers at a store say on "Sale-days."

(3) Probable results of an advertising scheme.

(4) Amount of reserves to be created to meet probabl conditions (*e.g.* of money, stocks, etc.).

The Zee Chart (or Z curves) is so called because it consists of three curves which somewhat resemble the letter Z. The curves plotted are (as shown below) :—

Sales Data

Month.	Jan.	Feb.	Mar	Apl.	May.	June.	July	Aug.	Sept.	Oct.	Nov.	Dec
Sales 1934	266	238	202	196	210	219	209	196	226	242	298	300
Cumulative	266	504	706	902	1112	1331	1540	1736	1962	2204	2502	2802
Moving Annual Total	—	—	—	—	—	—	—	—	—	—	—	—
Sales 1935	270	242	228	199	216	225	226	198	234	250	312	310
Cumulative	270	512	740	939	1155	1380	1606	1804	2038	2288	2600	2910
Moving Annual Total	2806	2810	2836	2839	2845	2851	2868	2870	2878	2886	2900	2910
Sales 1936	276	250	234	208	222	230	225	200	240	260	320	325
Cumulative	276	526	760	968	1190	1420	1645	1845	2085	2345	2665	2990
Moving Annual Total	2916	2924	2930	2939	2945	2950	2949	2951	2957	2967	2975	2990

FIG. 104.—DATA PREPARED FROM FIG. 91 COLUMN (b).

The moving annual total from end of 1934 is found by dropping January total 1934 and adding January 1935 and so on continuously through all succeeding years.

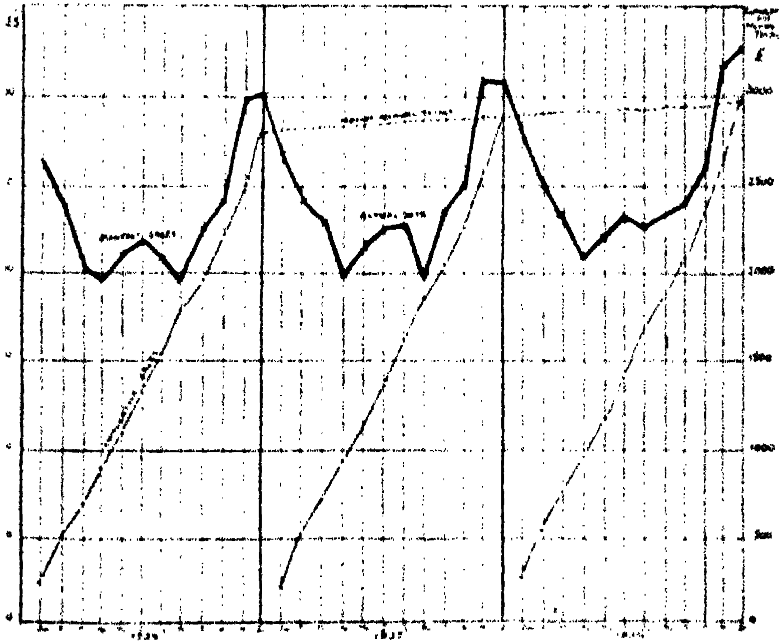

FIG. 105.—SALES CHARACTERISTICS SHOWN BY ZEE CHARTS.

(i) A curve of the original data (*e.g.* sales values or production quantities, etc.).

(ii) A curve of the cumulative data.

(iii) A moving total curve, which is plotted from a moving annual total if the data are monthly, 28-days, or weekly figures, or from a moving monthly total if the data are daily figures.

The scales used for the cumulative and moving total figures are usually about five or ten times that of the original data, to preserve proportions. With weekly figures the cumulative scale is better if twenty times the weekly scale.

The plotted curves. When two or more Zee Charts plotted for successive years are placed side by side the monthly and moving total curves respectively appear continuous from one chart to the next.

The cumulative curve always meets the moving total curve at end of each year.

The procedure is shown in Figs. 104 and 105.

Interpretation.—The heavy line in Fig. 105 shows the actual monthly figures and the fluctuations of a seasonal nature are seen. The cumulative curve, which is plotted month by month shows whether the year's sales total to date is more or less than that of the previous year. The moving average curve shows the trend both month by month and year by year.

The Galton Graph.—This graph, devised by Professor Francis Galton, is a practical means of determining the ratio of variation between two variables over a period of time, and is particularly useful when oscillating deviations are not all regular. For instance, the movement of, say, price, may be greater than that of supply. When considering correlation of two such variables the ratio of variation can be calculated by taking the deviation of the relative items (the supply) from the mean at each date and dividing this by the corresponding deviation of the subject (the price); the quotients thus obtained when added together and divided by the number of quotients, gives the desired ratio.

If a graph, as described below, is prepared a value of the ratio of variation is also obtainable by measuring the angle between a vertical line from the base line and the line of regression, *i.e.* Ratio of Variation = tan of the angle.

Instead of arithmetic calculation a Galton Graph may be used thus:—

(1) Convert the series of data into a series of index numbers, using the average of the items as the base (*i.e.* divide each item by the average multiplied by 100).

(2) Plot the indices of the subject on the vertical scale.

(3) Plot the indices of the relative on the horizontal scale.

(4) The points, although probably widely scattered, will show more or less as a band running up towards the right on the chart. Draw a line following the general trend as judged by the eye, but keeping an equal number of points on either side of the line, and as nearly as possible equidistant from it.

Example.—Indices, allowing one year's lag between manufactures and raw material production (hypothetical data), shown in Fig. 106 are plotted on the graph in Fig. 107.

Reading the Galton Graph.

(*a*) With perfect correlation every point would lie on a straight line (or on a well-defined curve when a lag exists between the series).

(*b*) If the line slopes to the left downward, the correlation is *direct.*

(*c*) If it slopes to the right downward, the correlation is *inverse.*

(*d*) If both series change by the same percentage, *e.g.* both increase or decrease by 3 per cent., the ratio of variation is obviously unity, and will be shown by the fact that the points plotted will so lie that the line drawn slopes at 45°—*i.e.* a **line of equal proportional variation.**

(*e*) When the relative changes less proportionately than the subject, there will be a different angle less than 45 per cent., and the line drawn is called the **Line of Regression.** The wider the divergence of the line of regression from the line at 45° the less the correlation, and *vice versa.*

(*f*) For reliability a large number of points should be plotted.

(*g*) When one series is affected by the other, say the following year (*i.e.* a lag of one year), the first series should commence one year later to allow for the lag, as shown in Fig. 106.

(*h*) To find the ratio of variation draw at any point a horizontal line which cuts through the line of regression, then the distance from the regression line intersection (*C*) to the ordinate axis (*B*), divided by the distance on the graph from *A* to *B* (the point where the regression line

Year.	Steel Manufactures.	Year.	Iron Ore Production.
1	130	2	100
2	130	3	130
3	160	4	165
4	140	5	110
5	120	6	130
6	100	7	110
7	80	8	90
8	65	9	70
9	100	10	90
10	90	11	90
11	125	12	95
12	90	13	103
13	65	14	95
14	60	15	80
15	85	16	70

FIG. 106.—DATA FOR GALTON GRAPH; THE SUBJECT IS STEEL MANUFACTURES; THE RELATIVE, IRON ORE PRODUCTION.

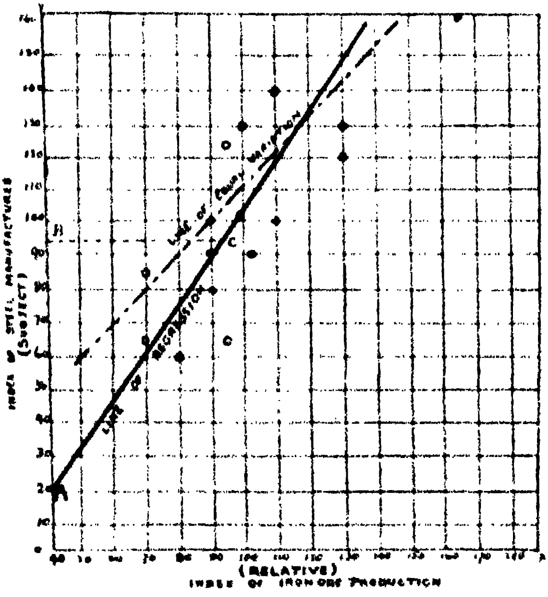

FIG. 107.—THE GALTON GRAPH SHOWING RATIO OF VARIATION OF TWO VARIABLES.

reaches the ordinate axis) gives the required ratio of variation. In the graph (Fig. 107) *BC* = 55, *BA* = 75 and the ratio of variation is therefore $\frac{55}{75} = 0{\cdot}73$. This informs us that for every change of 1 per cent. in the subject the relative tends to change 0·73 per cent.

The difference between the ratio and unity (*i.e.* 1–0·73 = 0·27) is called the **Ratio of Regression.**

STATISTICAL BUSINESS REPORTS

Tabulated Business Reports.—In view of the fact that the general procedure for the preparation of tabulated statements has been dealt with in an earlier chapter, and the arrangement of statistical reports for particular purposes is a matter for personal decision further description is unnecessary. The skeleton forms and suggestions for reports which follow will be found suitable for many kinds of business concerns.

The analysis of the financial position of a business as shown by the comparative balance sheet, *e.g.* Fig. 108, should include a report on :—

(*a*) The ratio of current assets to current liabilities, which gives a measure of the financial strength of the business when reviewed with other financial and trading facts, particularly the relationship between sales, sundry debtors, and stocks.

(*b*) The ratio between net worth (usually the capital account in a private business) and (i) total liabilities and (ii) fixed assets.

(*c*) The percentage gross profit and net profit on sales, together with comparative figures, as shown in Fig. 109.

(*d*) The comparative gross profit earned, by products or departments; and the general expenses and net profits of the business. A convenient form is given in Fig. 110.

For Management Purposes percentages or ratios should be calculated at regular convenient intervals to show :—

The return on capital.
The ratio of expenses to sales.
The stock turnover.
The turnover of trade debtors.

Comparative Statement of Financial Position

Liabilities.						Assets.					
Description. (a)	Last Year. (b)		This Year. (c)		Increase or Decrease.† (d)	Description. (e)	Last Year. (f)		This Year. (g)		Increase or Decrease.† (h)
	£	%	£	%	£		£	%	£	%	£
Floating Liabilities :—						*Floating Assets* :—					
Trade Creditors . .						Cash and Bank . .					
Expenses Accrued . .						Sundry Debtors . .					
Loan Creditors . .						Stock . .					
Other items (detailed) . .						Work in Progress .					
....						Prepaid Expenses .					
....						Other items . .					
Net Worth :—						*Investments* :—					
Share Capital						In Subsidiaries					
Preference . .						Stocks and Shares, etc. . . .					
Ordinary . .						Other items . .					
Reserves . . .						*Fixed Assets* :—					
Profit carried forward . .						Land . . .					
						Buildings . . .					
						Plant . .					
						Machinery . .					
						Motor cars .					
						*Patents . . .					
						*Goodwill . . .					
						Other items . .					

* Sometimes separately totalled as " Intangible Assets."   † Decreases usually entered in red.

FIG. 108.—TABLE SHOWING COMPARATIVE ANALYSIS OF FINANCIAL POSITION OF A BUSINESS.

These are required in addition to the various statements in a manufacturing business, to show the efficiencies of labour (see Fig. 5), machines and production.

Production management may be well served with Gantt Charts (see Figs. 27, 28). When applied to labour the chart shows how far towards the fullest advantage use has been made of each worker; and by suitable symbols delays, lost time and the reasons therefor can be indicated. Applied to machine times and output, every stoppage or delay can be similarly indicated. Markings can be inserted to show starting times, scheduled finishing times, the causes of stoppages, the actual output as compared with the scheduled (Fig. 28). Similarly machine layouts, loads and progress of production can be easily charted and kept under review.

Sales Analyses. (*a*) **Comparisons of Monetary Values and Quantities.**—These are important statistics. When prices of articles are increasing with the fall in the purchasing power of money, it will be apparent that if the total value of sales is maintained at a stationary level, the *quantity* of articles sold will be less. To show progress, quantities sold would have to increase as well as turnover in value. The easiest way to review this comparison is by a chart on which are plotted curves for sales values and sales quantities, using two scales for the vertical readings so arranged that the unit value of a unit quantity are each represented by the same vertical distance. The relative movements of the two curves thus shown will give a proper comparison and a clear indication of the trading position. The money values may be shown on the left side of the chart, the quantities (weight, numbers, etc.) on the right, and the dates or periods on the horizontal axis.

(*b*) **Comparison between Sales and Purchases** can be effectively shown on a graph as a means of keeping a check on the stock position.

(*c*) **Salesmen's Calls and Orders.**—The number of calls made and the number of orders taken by each salesman may be plotted on a ratio-ruled chart. The ratio of orders to calls made should usually tend to increase in a progressive business.

Similarly the value of business secured by salesmen should be plotted as a curve on ratio-ruled paper, to obtain a proper indication of the trend of each man's selling results.

When a quota has been set for a sales manager or salesman, *Comparative Summary of Revenue and Expenditure*

Details.	Last Year.		This Year.		Increase or Decrease.	
(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)
	£	%	£	%	£	%
Gross Sales						
Less Returns and allowances .						
Net Sales						
Less Prime Cost of Sales * .						
Gross Profit on Sales . . .						
Less Selling Expenses . .						
Less General Expenses .						
Net Profit on Sales . . .						
Add Other Income (net) . .						
....						
....						
Net Income						
Less Fixed Charges . . .						
Add Balance brought forward from last year						
Amount available . . .						
Proposed appropriation :						
Transfer to Reserves . .						
Dividends (net) . . .						
Income Tax						
To carry forward . . .						

* In a manufacturing business this would be subject to deduction also of "Works Expenses," hence substitute the words "Production Cost."

N.B.—(1) When only one type of product is sold an additional entry, viz. "Quantity Sold" may be made to report the number of units sold in both years; if more than one type a separate report is better.

(2) The upper part may be presented as a monthly (or other period) report.

FIG. 109.—A TABLE GIVING USEFUL INFORMATION FOR MANAGEMENT PURPOSES.

the extent to which results fall short of, or exceed, the quota may be usefully shown by drawing a line horizontal to the base line at the quota level on the vertical scale, and plotting the actual results. When the curve falls below the quota line, the fact that sales have not reached the scheduled figure will be readily seen.

Useful Information which may be the subject of statistical reports, either graphical or tabular —

Advertising.

(*a*) The relation between inquiries and orders shown on a ratio chart. A drop in effective orders may be due to unsatisfactory advertising, or to inefficient handling of inquiries.

(*b*) The relation between advertising and sales. A ratio chart serves to measure the effectiveness of the advertising.

(*c*) The probable sales result of an advertising campaign may be estimated by use of a trend chart on probability paper.

Sales Department.

(*a*) Comparative sales of departments or products.

(*b*) Comparison between actual sales and the sales quota, such as for total sales, departmental sales, individual areas, individual travellers.

(*c*) Travellers' Reports, shown on charts or by tables, viz. calls made, orders taken, value and number, prospective orders, average cost per order, or percentage cost, ratio of orders to calls.

(*d*) Comparative distribution of sales for several years, as shown by a simple frequency chart, or a cumulative frequency chart (ogive).

(*e*) A Zee Chart for annual sales shows at a glance (i) how sales are progressing week by week, or month by month, etc.; (ii) how the cumulative yearly performance to date compares with that of the preceding year or years; (iii) the progress of sales upon an annual basis with the seasonal fluctuations eliminated (see Fig. 105).

(*f*) Trend charts can be used for forecasting and reviewing the long period indications.

(*g*) Comparative sales totals, gross and net profits, as shown, for instance in Figs. 109 and 110.

Comparative Profit and Loss Summary
Month ended..............19...

	Monthly Totals.				Cumulative Totals.			
	Last Year.*		Current Year.		Last Year.		Current Year.	
(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)	(*g*)	(*h*)	(*i*)
	Details.	Totals.	Details.	Totals.	Details.	Totals.	Details.	Totals.
	£	£	£	£	£	£	£	£
Gross Profit :—								
Dept. or Product 1 . .								
,, ,, 2 . .								
,, ,, etc. . .								
Total								
***Deduct* Expenses** :—								
(Detail under main headings in columns (*b*), (*d*), (*f*) and (*h*)) .								
....								
....								
....								
Net Trading Profit . . .	—		—		—		—	
Add **other profits** :—								
....								
....								

* The comparative figure may be the preceding month of the same year in some instances.

FIG. 110.—A TABLE GIVING COMPARATIVE MONTHLY TRADING RESULTS, AND THE CUMULATIVE COMPARATIVE FIGURES FOR THE YEAR.

Debt Collection Management.

(*a*) Chart showing deliveries invoiced to customers and the payments received. For important customers a separate chart can be used.

(*b*) Efficiency of department as shown by a curve of total sales on credit and one of accounts uncollected. On ratio paper any rate of increase in uncollected accounts would be indicated and call for inquiry.

Purchasing Department.

(*a*) The fluctuations and trend in prices of important purchases.

(*b*) The position of deliveries against bulk contract supplies.

(*c*) The relationship between Purchases, Stocks and Sales.

QUESTIONS

1. Explain briefly and concisely the following: (*a*) Probable Error; (*b*) An Array; (*c*) Ogive; (*d*) Geometric Progression; (*e*) Historigram. (*London Association of Certified Accountants.*)

2. Prepare a form of financial statement for submission to the directors of a limited company showing at a glance the trend of business during the preceding month. (*London Association of Certified Accountants.*)

3. Define: (*a*) Histogram; (*b*) Historigram; (*c*) Continuous Series; (*d*) Discrete Series. (*London Association of Certified Accountants.*)

4. What are the characteristics of the coefficient of dispersion and in what respects may it be used with advantage? (*London Association of Certified Accountants.*)

5. Give four of the commoner types of diagrams in use and briefly explain the advantages and disadvantages of each type chosen. (*London Association of Certified Accountants.*)

6. Using hypothetical figures construct an Ogive, and state briefly the principal uses of such a curve. (*London Association of Certified Accountants.*)

7. The sales statistics of a manufacturer show the following tonnages for the past three years. Bring out in a graph the salient characteristics.

Year.	Month.											
	Jan.	Feb.	Mar.	Apl.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1932 . . .	147	186	210	240	200	170	190	186	142	131	140	164
1933 . . .	182	204	200	271	239	216	160	155	184	195	210	231
1934 . . .	175	169	166	254	261	266	210	212	198	217	231	252

(*Incorporated Accountants (Final).*)

8. Define Histogram, Ogive, and Frequency Polygon. (*Incorporated Accountants* (*Final*).)

9. Assuming that all the statistics you required were available, what figures would you use, and in which way would you use them, to determine whether an advertising campaign in the Daily Press of an article in daily household use had been planned upon the most economic and effective basis? (*Incorporated Accountants* (*Final*).)

10. Draw a histogram showing, on the same basis, the comparative wages in States A, B and C:—

Wages per Week ($).		No. of Wage Earners in State		
Exceeding.	Not Exceeding.	A.	B.	C.
2·00	3·09	165	264	398
4·00	5·99	380	1656	1010
6·00	7·99	502	2490	1617
8·00	9·99	1330	1215	2203
10·00	11·99	1740	806	2842
12·00	13·99	1620	1721	3396
14·00	15·99	1325	3512	3482
16·00	17·99	1095	3496	3144
18·00	19·99	800	1787	1498
20·00	21·99	500	1224	354
22·00	23·99	378	796	56
24·00	25·99	95	620	...
26·00	27·99	70	320	...

(*Incorporated Accountants* (*Final*).)

11. Given the data below, how would you compare the total production in 1907 with that in 1894?

Production of Worsted Tissues

	1894 (yards). (000's).	1907 (yards). (000's).	Mean Price, 1894–1907 (*d.* per yard).
Broad coatings, all wool .	1,117	1,379	46·2
„ „ mixed .	385	720	28·0
Narrow coatings, all wool .	217	41	31·7
„ „ mixed .	272	159	20·2
Stuffs, all wool . . .	1,320	1,043	11·9
„ mixed . . .	7,756	6,559	9·4
Total of Worsted Tissues .	11,067	9,901	—
Ratio . . .	100	89·4	
Total value . . .	£6,666,000	£7,394,000	
Ratio . . .	100	110·9	

(*Incorporated Accountants* (*Final*).)

12. Graph the following data in the most effectively comparative form :—

Comparative Wages in States A, B and C.

Wages per Week ($).	State A. No. of Wage Earners.	State B. No. of Wage Earners.	State C. No. of Wage Earners.
4·00— 5·99	3,784	16,424	10,102
6·00— 7·99	5,025	24,898	17,170
8·00— 9·99	13,200	12,122	22,054
10·00—11·99	17,420	8,964	28,402
12·00—13·99	16,142	17,220	33,960
14·00—15·99	13,240	35,116	34,817
16·00—17·99	10,940	34,963	31,460
18·00—19·99	7,964	17,842	24,972
20·00—21·99	4,982	12,240	3,417

(*Incorporated Accountants* (*Final*).)

13. From the following details construct a diagram, showing the monthly turnover of the A. B. Co., Ltd., for the three years ended December 31, 1925 :—

	1923.	1924.	1925.
	£	£	£
January	3250	2400	2000
February	3100	3000	2800
March	3450	3100	2900
April	3600	3300	2850
May	3580	2800	3000
June	4000	3250	3000
July	4000	3300	3550
August	3900	3400	3700
September . . .	4200	3250	4000
October	4250	3800	4100
November . . .	3750	3750	3800
December . . .	4000	3600	3900

(*London Association of Certified Accountants.*)

14. Define the following terms : Ogive ; Frequency Graph ; Index Number ; Mode ; Standard Deviation. (*London Association of Certified Accountants.*)

15. What rules would you obey in smoothing a frequency polygon ? (*Incorporated Accountants* (*Final*).)

16. What is a Galton Graph ? State how it is constructed and interpreted. (*Incorporated Accountants* (*Final*).)

17. Draw an illustrative diagram showing what facts you would graph on *one* chart if you were asked to show monthly thereon, over a period of three years, the maximum of useful information in regard to the sales of one product. (*Incorporated Accountants* (*Final*).)

18. Compute 3- and 5-year moving averages for the following indexes relating to the period 1910–1920:—

100, 95, 108, 100, 98, 106, 106, 109, 108, 99, 103.

Plot the data and both moving averages on the same chart. (*Building Societies Institute.*)

19. Draw a graph from the following figures, illustrating the distribution of sex and age of the population:

Population of England and Wales, 1921, *by Sex and Age*
(in thousands).

Age Last Birthday.	Males.	Females.
Under 5 years	1,681	1,640
5 and under 10 years	1,767	1,752
10 .. 15 ..	1,837	1,823
15 .. 20 ..	1,728	1,775
20 .. 25 ..	1,418	1,703
25 .. 30 ..	1,340	1,620
30 .. 35 ..	1,281	1,520
35 .. 40 ..	1,273	1,472
40 .. 45 ..	1,223	1,378
45 .. 50 ..	1,162	1,244
50 .. 55 ..	971	1,043
55 .. 60 ..	782	849
60 .. 65 ..	601	681
65 .. 70 ..	441	537
70 .. 75 ..	280	376
75 .. 80 ..	159	234
80 .. 85 ..	70	113
85 years and over	25	51

(*N.A.L.G.O.*)

CHAPTER XXII

LOGARITHMS AND THEIR USE

Logarithms Simplify Calculations.—If the logarithms of numbers are substituted for actual numbers, the tediousness of multiplication, division, squaring and raising to various powers, extracting of square, cube and other roots is greatly reduced.

Once the method is understood—and it is quite simple—logarithmic calculations are very easy.

What are Logarithms?—Natural numbers can be stated as powers of 10. Logarithms of the numbers are the exponents of the powers. This will be understood better from examples :—

$100 = 10^2$ therefore the logarithm of 100 is 2
$1000 = 10^3$,, ,, 1000 is 3
$10 = 10^1$,, ,, 10 is 1
$1 = 10^0$,, ,, 1 is 0

When a number is less than 1, the logarithm is negative. Thus for

0·001 the logarithm is -3
0·0001 ,, ,, -4
0·01 ,, ,, -2
0·1 ,, ,, -1

It will be seen that the logarithms of the above numbers are all integral numbers. The logarithms of all other numbers which are not multiples of 10 are fractions.

It will be seen therefore that :—

For any number between 1 and 10 the log[*] is between 0 and 1.
For any number between 10 and 100 the log is between 1 and 2, and so on.
For any number between 1 and 0·1 the log is between 0 and -1.

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[*] It is usual to write "logarithm of" as log. Thus we say log 100 = 2, and so on.

For any number between 0·1 and 0·01 the log is between − 1 and − 2, and so on.

Any fractional log is *always* treated as positive, but as a number less than 1 has a negative log. the log has to be written in such a way that the fractional part is kept positive. How this is done will be explained presently.

How to Write a Logarithm.

A logarithm consists of two parts :—

(*a*) An integral number like those quoted above, which is called the *characteristic*, which may be + or −.

(*b*) A fractional part called the *Mantissa*, which is always +.

To illustrate this look for the log of 209 in a table of logarithms. We find opposite the number 209·0 the figures 32,015. Now the characteristic of any number is one *less* than the number of digits to the left of the decimal point. 209 has three digits, therefore the characteristic is 2, and we write the log of 209 as 2·32015.

Now examine the number 0·2090. The number is less than 1, therefore the characteristic is negative, and it is one *more* than the number of zeros between the decimal point and the first significant figure of the number. In 0·2090 there is no zero between the decimal point and the first figure (2), hence the characteristic is one more than zero, and is negative, *i.e.* − 1, so we write the log thus : $\bar{1}$·32015 (not − 1·32015 because only the characteristic is negative).

Take another series of examples :—

Log 6436 = 3·80862
Log 643·6 = 2·80862
Log 64·36 = 1·80862
Log 6·436 = 0·80862
Log 0·6436 = $\bar{1}$·80862
Log 0·006436 = 3·80862
Log 0·0006436 = $\bar{4}$·80862 and so on.

How to Find a Number from its Logarithm.—Suppose we have the logarithm 2·63215 and we want the number it represents. In the tables we turn to the portion called antilogarithms, and looking down the mantissæ we find opposite 63215 the number 4287.

In the logarithm 2·63215 the characteristic (2) is positive, hence the number the log represents must have three digits, so we place our decimal point accordingly, viz. 428·7.

If the log had been $\bar{2}$·63215, the number represented by the log must have one zero between the decimal point in the number and the first significant figure; hence we place our decimal point 0·04287; this is the number represented by log $\bar{2}$·63215.

The following further examples will assist :—

Logarithm.		Number.
3·63215	=	4287·0
5·03215	=	428700·0
1·63215	=	42·87
0·63215	=	4·287
$\bar{4}$·63215	=	·0004287
1·63215	=	·4287
$\bar{3}$·63215	=	·004287

Computations by Logarithms

The following rules are all that it is necessary to know :—

To multiply numbers find their logs and *add* them together. Look up in the table of antilogarithms to find the number corresponding to the total, insert the decimal point and you have the required product.

Example (i)—Multiply 64·36 × 42·87.

Add the logs of these numbers :—

Log 64·36 = 1·80862
Log 42·87 = 1·63215

Log of product = 3·44077

In the table of antilogarithms 44077 gives the number 27,591

∴ product = 2759·1.

Example (ii)—Multiply 6436 × 0·04287.

Log ·04287 = $\bar{2}$·63215, but for ease of adding or subtracting a logarithm with a negative characteristic we add and subtract 10 to make a positive characteristic, thus :—

Log ·04287 = 8·63215 − 10
Log 6436 = 3·80862

Log of product = 12·44077 − 10
= 2·44077

Antilog of 44077 = 27591
∴ product = 275·91.

To Divide Numbers.

Subtract the logarithm of the divisor from the log of the dividend. The difference is the log of the quotient :—

Example—Divide 161·2 by 37·49.

Log 161·2 = 2·20729
Log 37·49 = 1·57392

Log of quotient = 0·63337
Antilog of 63337 = 4299
∴ quotient = 4·299

To Extract the Root of a Number.

Divide the logarithm of the number by the index of the root; the quotient is the log of the desired root.

Example (i)—Find the square root of 183,784.

Log 183784 = 5·26430
Divide by 2 2)5·26430
Log of square root = 2·63215
Antilog of 63215 = 4287
∴ The square root is 428·7.

Example (ii)—Find the 5th root of 84140.

Log 84140 = 4·92500
Divide by 5 5)4·92500
·98500
Antilog of 98500 = 9661
∴ The 5th root is 9·661.

To Raise a Number to a Stated Power.

Multiply the logarithm of the number by the exponent of the power; the product is the log of the required power.

Example.—Find the cube of 90.

Log 90 = 1·95424
Multiply by 3 = 5·86272
Antilog of 86272 = 7290
∴ 289 cubed = 729,000

To Find the Geometric Mean.—The geometric mean of 225,

475, 699, 5913, 29840 is $\sqrt[5]{225 \times 475 \times 699 \times 5913 \times 29{,}840}$. This can be solved easily by logarithms :—

Log 225	=	2·35218
,, 475	=	2·67669
,, 699	=	2·84448
,, 5913	=	3·77181
,, 29840	=	4·47480
		16·11996
Divide by 5		5)16·11996
		3·22399
Antilog of 22399	=	167490
∴ Geometric mean	=	1674·9

QUESTIONS

1. Multiply 429·9 by 374·9, and 3749 × 0·0429.
2. Divide 78,240 by 67·50, and find the 12th root of 78,240.
3. Cube 485 and divide the result by 25·5.
4. Find the geometric mean of 312, 418, 3715, 3621, 487, 298.
5. What numbers are represented by the following logarithms ?

3·75967, $\bar{2}$·75967, 0·81291, $\bar{4}$·79309
$\bar{1}$·99782, 6·99782, 8·75967, 4·79309

6. Find the following :—

Cube root of 6672.	Square of 599·8.
Square root of 183784.	Cube of 0·05236.
Sixth root of 36721942.	15th root of 8641000.

7. If log 15 is 1·17609 write logarithms of the following numbers :—

15,000	1·5	1500·0
150,000	0·15	0·015
150	0·0015	0·0015

APPENDIX

SIGNS COMMONLY USED IN STATISTICS

Σ (or $\int$) means the summation of - -.

Thus $\Sigma(x^3) = 1^3 + 2^3 + 3^3 + \ldots$

x is usually used to denote a variable, its individual values being indicated by $x_1\, x_2\, x_3\, \ldots\, x_n$.

n = the number of items.

a = arithmetic average (also sometimes written $\bar{x}$).

w is usually used for "weights," hence $w_1\, w_2\, \ldots\, w_n$ are weights for individual items. In a frequency distribution f or v may be used instead of w.

f = frequency. f_0, f_1 and f_2 are the frequencies in the groups of which the modal group forms the centre.

ξ = distance from average.

$\mathbf{g}$ (g) = geometric mean or average $(g = \sqrt[n]{x_1 \times x_2 \times \ldots x_n})$.

m = items in a group or series.

M = median.

Z = mode.

c = class range or interval.

l_1 and l_2 = limits of a modal group or of the class containing the median.

d = deviation from arithmetic mean $(x - a)$.

d_m = deviation from median $(x - M)$.

d_z = deviation from mode $(x - Z)$.

δ_a, δ_m, δ_z = mean or average deviation.

σ = Standard deviation $= \sqrt{\dfrac{\Sigma(x - a)^2}{n}}$.

σ^2 = variance.

Q_1 and Q_3 = Lower Quartile and Upper Quartile (the median (M) lies between).

$Q.D.$ = Quartile Deviation (semi-inter-quartile range).

V = Coefficient of Variation $\left(\dfrac{100\sigma}{a}\right)$.

j_1, j_2, j_3 = First, second and third coefficients of skewness.

e = absolute error.

ϵ = relative error.

$P.E.$ = Possible error.

r = Coefficient of correlation (or co-variation).

Δ = difference.

THE LONDON CHAMBER OF COMMERCE
(INCORPORATED.)

SPRING EXAMINATION, 1936

FOR

HIGHER COMMERCIAL EDUCATION CERTIFICATES

Monday, May 25th—*7 to 10 p.m.*

BUSINESS STATISTICS

Examiner—PROF. R. B. FORRESTER, M.A., M.COM.

INSTRUCTIONS TO CANDIDATES.

Six questions to be answered, of which one must be either Question 1 or 2.

1. The Table shows the Distribution of Wage Earners in a Company by weekly earnings.

July, 1933

No. of Wage Earners.	Weekly Earnings	
	over.	not exceeding.
250	10*s*.	20*s*.
700	20*s*.	30*s*.
1260	30*s*.	40*s*.
1853	40*s*.	50*s*.
2700	50*s*.	60*s*.
3048	60*s*.	70*s*.
1963	70*s*.	80*s*.
750	80*s*.	90*s*.
200	90*s*.	100*s*.

From the above table obtain by graphical methods (*a*) the median; (*b*) the upper and lower quartiles. From these calculate the quartile coefficient of dispersion.

2. The price index-numbers of ten foodstuffs for two years 1913 and 1930 are:—

	1	2	3	4	5	6	7	8	9	10
1913	57	65	66	70	72	73	67	81	92	97
1930	77	91	84	91	95	112	95	143	120	132

Find a measure of the correlation between these two sets of indices.

3. Give an account of the official method of measuring changes in the retail prices of working class "necessaries of life."

4. What do you understand by dispersion, and what are the different ways of measuring it?

5. Explain what is meant by the following terms :—

(*a*) Unemployed person.
(*b*) Imports.
(*c*) Raw Materials and Articles mainly unmanufactured.
(*d*) Shipping entered and cleared with cargoes.

6. Discuss the importance of exact definition in statistics. Illustrate your answer by reference to either (*a*) occupational statistics; or (*b*) statistics of the Balance of Trade.

7. Write notes upon two of the following :—

(*a*) Simple and Compound Bar Charts.
(*b*) Frequency Charts.
(*c*) Z Charts.

8. Give some account of the British official statistics relating to transport.

9. Show how the *per capita* consumption of commodities can be used to help in sales investigations.

10. Discuss the forms of return and reports which would be of value in assessing the financial position of a company. How far would they cast light upon the efficiency of the management?

THE LONDON ASSOCIATION OF CERTIFIED ACCOUNTANTS

(LIMITED BY GUARANTEE.)

FINAL EXAMINATION, JUNE 1936

SECTION II

PRACTICAL ACCOUNTING AND STATISTICS

PART B.

1. An estimate of the population of a town is given as 49,000 whereas actually the population is 50,000.

Compute (*a*) the Absolute Error; (*b*) the Relative Error; and (*c*) the Percentage Error.

2. State the chief advantages of a logarithmic scale graph as compared with a natural scale graph.

3. Calculate the standard deviation from the following data, using the short-cut method :—

Exceeding.	Not Exceeding.	*f*.
7½	8½	2
8½	9½	4
9½	10½	6
10½	11½	7
11½	12½	9
12½	13½	3
13½	14½	1

UNION OF LANCASHIRE AND CHESHIRE INSTITUTES

Advanced Commercial Course

STATISTICS

Monday, April 8th, 1935—7 to 10 p.m.

Six *questions only are to be answered. Your selection must include* at least one *of the starred questions.*

Squared Paper and Mathematical Tables are supplied. A Slide Rule and Tables of Squares, etc., may be used.

1. The numbers of a certain class of workpeople receiving different wages are as follows :—

Wages	*20s.-20s. 6d.*	*20s. 6d.-21s.*	*21s.-21s. 6d.*	*21s. 6d.-22s.*	
No. of workers	3	7	12	10	
Wages	*22s.-22s. 6d.*	*22s. 6d.-23s.*	*23s.-23s. 6d.*	*23s. 6d.-24s.*	
No. of workers	34	50	52	103	
Wages	*24s.-24s. 6d.*	*24s. 6d.-25s.*	*25s.-25s. 6d.*	*25s. 6d.-26s.*	*26s.-26s. 6d.*
No. of workers	68	32	10	16	3

Re-tabulate these data in four groups, expressing the number of workers in each group as a percentage of the whole.

2. The estimated expenses of an industry consist of £487,000 for wages and £216,000 for other expenses; and the estimated receipts from sales are £750,000. Calculate the estimated net profits, giving the limits of possible error on the assumption that the amounts stated are liable to errors of 2 per cent., 8 per cent., and 9 per cent. respectively.

3. Find the average of the items 71·9, 83·7, 52·6, 97·3, 39·9, 72·0 when weighted with weights proportional to 2, 5, 1·5, 7, 4, 3.

What would be the approximate effect on the result if all the weights were increased by 0·5?

4. The number of people employed in a certain industry decreased from 10,820 to 6745 between the years 1900 and 1930. Estimate the number employed in 1920 on the two assumptions that the yearly numbers formed (*a*) an arithmetic progression, and (*b*) a geometric progression.

5. Find the arithmetic mean of the wages of the workpeople given in Question 1.

6. Draw graphs to illustrate the variation in numbers of

students in cotton weaving and cotton spinning, as given in the table :—

Year.	1923.	1924.	1925.	1926.	1927.	1928.
Weaving students .	2300	2500	2800	3100	3200	3500
Spinning students .	3000	3200	3300	3500	3100	2800

Year.	1929.	1930.	1931.	1932.	1933.	1934.
Weaving students .	3400	3800	2700	1800	1400	1300
Spinning students .	2600	2500	2000	1400	1100	800

7. Draw a block diagram (histogram) to represent the statistics of Question 1, and use the diagram to estimate roughly the median and mode of the distribution.

8. Calculate the standard deviation and the quartile deviation of the wage distribution of Question 1.

*9. The numbers of persons injured in road accidents in Manchester during each of the thirty weeks preceding November 3rd, 1934, were : 72, 66, 47, 68, 64, 52, 45, 64, 55, 50, 49, 58, 52, 79, 59, 70, 82, 52, 60, 60, 84, 53, 52, 58, 53, 53, 49, 57, 63, 67. Use these figures to explain the method of moving averages to draw graphs showing the general trend and the short-time fluctuations. (Use five-weekly averages.)

*10. During the same thirty weeks as those covered in the statistics of Question 9, the numbers of persons killed in road accidents in the whole of Great Britain were : 118, 133, 126, 124, 128, 127, 146, 134, 153, 142, 134, 139, 180, 144, 144, 156, 149, 160, 156, 159, 142, 160, 159, 161, 130, 144, 131, 153, 168, 178. Calculate approximately the coefficient of correlation between these two sets of figures.

ROYAL SOCIETY OF ARTS EXAMINATIONS FOR ADMINISTRATIVE AND OPERATIVE STAFFS OF ROAD TRANSPORT UNDERTAKINGS

Group III

ELEMENTARY STATISTICS

Tuesday, May 26th, 1936

[*Three hours allowed.*]

Not more than SEVEN *questions to be attempted.*

Answer both questions in Section A, three questions in Section B, the question in Section C, and one question in Section D.

Section A

Answer BOTH *questions*

1. If you were asked to conduct a statistical inquiry into the traffics of a bus undertaking with a view to ascertaining whether the average receipt per passenger and the average distance travelled per passenger were increasing, describe briefly (*a*) the basic data you would require, (*b*) what steps you would take to assemble the data, and (*c*) what methods you would employ to analyse such data.

2. It is required to estimate the population of a large town. The last census was taken five years ago. What methods do you suggest?

Section B

Answer THREE *questions only*

3. How would you distinguish :—

(i) The simple average.
(ii) The weighted average.
(iii) The median?

4. Dealings in a certain security at the following prices took place on the Stock Exchange :—

$100\frac{5}{16}$ $100\frac{3}{8}$ $100\frac{1}{4}$ $100\frac{5}{16}$ $100\frac{7}{16}$ $100\frac{1}{4}$ $100\frac{3}{8}$ $100\frac{7}{32}$ $100\frac{11}{32}$
$100\frac{3}{16}$ $100\frac{1}{8}$ 100 $99\frac{7}{8}$ $99\frac{27}{32}$ $99\frac{13}{16}$ $99\frac{3}{4}$ $99\frac{1}{4}$

Find :—

(i) The average price.

(ii) The median price.

5. Find the mean, mean deviation, mode and quartiles of the Stock Exchange prices quoted in Question 4.

6. If average money wages rise 20 per cent., and the retail purchasing power of money rises 10 per cent., by how much do average real wages rise?

7. Crude birth and marriage rates are commonly calculated as so much per 1000 of the population. Criticise this method. Can you suggest a better?

SECTION C

Answer this question

8. Draw a rough bar diagram (a histogram) illustrating the wage data shown below. Would a continuous curve have been more appropriate? If not, state reasons.

Grade.	Weekly rates of Wages at							
	July, 1914.		Dec. 31st, 1924.		Dec. 31st, 1928.		Dec. 31st, 1930.	
	s.	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*
Fitters . .	38	11	56	6	58	9	59	1
Ironmoulders .	41	8	60	0	62	1	62	4
Labourers . .	22	10	40	1	41	11	42	1

SECTION D

Answer ONE *question only*

9. What statistical units would you employ to compare the relative operating costs of two road passenger transport undertakings? Give your reasons.

10. Upon what basic data would you calculate the following statistics used by transport undertakings :—

(i) Operating ratio.

(ii) Average scheduled speed in service—m.p.h.

(iii) Average number of service car miles run per crew per duty?

11. What is the object of the calculation of a retail Price Index? Explain the method of calculation of a typical index.

THE SOCIETY OF INCORPORATED ACCOUNTANTS AND AUDITORS

FINAL EXAMINATION

November, 1935

(*Time allowed—one-and-a-half hours*)

IV.—STATISTICAL METHODS

(*Candidates should attempt* FIVE *questions only*)

1. Examine the characteristics and purposes of classification in statistical methods.

2. State the considerations you would observe in constructing a statistical table.

3. Arrange the following data in a frequency distribution. Calculate the arithmetic average, median and mode.

Statistics of Wages of Weekly Wage-Earners

Weekly Wage. (1)		No. of Wage-Earners. (2)	Weekly Wage. (1)		No. of Wage-Earners. (2)
s.	*d.*		*s.*	*d.*	
14	0	1	28	0	1
15	0	1	29	0	1
18	0	4	30	0	10
19	0	2	31	0	1
20	0	7	32	0	1
20	6	1	32	6	1
21	0	4	35	0	1
22	0	4	36	0	1
23	0	2	38	0	1
24	0	8	40	0	3
25	0	7	45	0	6
25	6	1	50	0	1
27	0	1	55	0	1
			Total		72

4. What rules would you obey in smoothing a frequency polygon?

5. State the advantages and disadvantages of the arithmetic average.

6. Calculate to two decimal places the standard deviation of the following frequency distribution :—

m.	*f.*	*m.*	*f.*
15	2	40	4
20	22	45	6
25	19	50	1
30	14	55	1
35	3		

7. Graph the short-time oscillations of the following price indices, using a half-yearly base :—

Month.	Index.	Month.	Index.
1	80	16	96
2	82	17	98
3	86	18	106
4	91	19	114
5	83	20	112
6	85	21	109
7	89	22	106
8	96	23	112
9	93	24	120
10	90	25	118
11	91	26	112
12	94	27	110
13	100	28	107
14	105	29	113
15	102	30	115

8. What is a Galton Graph? State how it is constructed and interpreted.

9. Mr. X is ordered abroad for his health for three years and gives you a power of attorney to manage his three drapery businesses situated in different parts of London.

Sketch roughly the form of statistical chart or charts that you would ask each local manager to provide.

THE SOCIETY OF INCORPORATED ACCOUNTANTS AND AUDITORS

FINAL EXAMINATION

May, 1936

(*Time allowed—one-and-a-half hours*)

IV.—STATISTICAL METHODS

(*Candidates should attempt* FIVE *questions only*)

1. Why is statistical science necessary, and what are its uses?

2. Classify the methods of statistical investigation, and state briefly their respective merits and demerits.

3. Work out (showing your workings) the most absolutely accurate result of multiplying *726* by *10,2*00 when the digits in italics only are accurate.

4. Compute to two places of decimals the standard deviation of :—

m. Mid-Point.	*f.*	*m.* Mid-Point.	*f.*
6	2	10	11
7	4	11	10
8	6	12	9
9	12	13	8

5. State the advantages and disadvantages of the median.

6. Draw a histogram showing, on the same basis, the comparative wages in States A, B and C :

Wages per Week ($).		No. of Wage Earners in State.		
Exceeding.	Not Exceeding.	A.	B.	C.
2.00	3.99	165	264	398
4.00	5.99	380	1656	1010
6.00	7.99	502	2490	1617
8.00	9.99	1330	1215	2203
10.00	11.99	1740	896	2842
12.00	13.99	1620	1724	3396
14.00	15.99	1325	3512	3482
16.00	17.99	1095	3496	3144
18.00	19.99	800	1787	1498
20.00	21.99	500	1224	354
22.00	23.99	378	796	56
24.00	25.99	95	620	—
26.00	27.99	70	320	—

7. A company manufacturing some 700 different products desires to take out significant statistics of its sales in the United Kingdom, using a numerical code for each characteristic selected, and performing the operations of classification by the use of Hollerith machines. The Hollerith card contains 45 columns of figures, each running downwards vertically from 1 to 9.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9

State the characteristics you would select, and show how you would use the columns on the card to the best advantage.

8. Define skewness. State the formulæ for the various coefficients of skewness, adding notes on their respective advantages and disadvantages.

Use the following symbols :—

a = arithmetic average,
M = median,
Z = mode,
Q = quartile,
d = deviation.

THE INSTITUTE OF TRANSPORT

GRADUATESHIP EXAMINATION

Thursday, April 27th, 1939

(*Time—2.30 p.m. to 5.30 p.m.*)

ELEMENTS OF STATISTICS

(Six—*but not more than six—questions should be attempted*)

1. Estimate from each of the following items the value of total imports, showing the limits of estimation :—

India Imports, April to September, 1938

Imports from :	Amount (Lakhs of Rupees).	Per Cent. of Total Imports.
United Kingdom . .	22,50	31·8
Total British Empire . .	42,43	60·1
Japan	6,15	8·7
Germany	5,87	8·3
U.S.A.	4,67	6·6

2. Compute the average of each of the following distributions :—

Age Distributions of British Born and Foreign Born Males in Canada, 1931

(*Percentages*)

Ages :	Under 15	15–	20–	25	30–	35–	40–
British . .	4·92	4·35	8·02	9·66	9·20	9·96	12·07
Foreign .	7·06	3·55	7·97	12·67	13·09	12·91	11·98

Ages :	45–	50–	55–	60–	65 and over	Total.
British . .	12·49	9·97	6·50	4·96	7·90	100
Foreign .	10·19	7·28	4·91	3·35	5·02	100

3. From the data of Question 2 make cumulative tables for the ages of British-born and Foreign-born males in Canada in 1931, and draw cumulative graphs for them. From the graphs estimate the median and quartile ages.

4. Compute any percentages or ratios which you think would assist in a better understanding of the following table :—

Gross Earnings of British Shipping in Foreign Trade, 1931

Kind of Trade.	Gross Receipts (£000).	Expenses Abroad (£000).	Net Receipts (£000).
U.K. and Empire Countries .	27·9	10·6	17·3
Empire Countries . . .	7·7	4·8	2·9
U.K. and Foreign Countries .	21·2	7·4	13·8
Empire Countries and Foreign Countries	8·5	5·0	3·5
Foreign Countries . . .	7·8	4·1	3·7
Totals . . .	73·1	31·9	41·2

5. Compute from the following data the average amount of sales per shop for the whole group of businesses covered :—

Retail Trade, Eire, 1933

Size of Business.	Percentage Number of Shops.	Average Number of Persons per Shop.	Average Sales per Person (£).
£100 and less than £500	40·5	2·0	134
£500 .. .. £1,000	20·6	2·6	266
£1,000 .. .. £2,500	21·1	3·5	445
£2,500 .. .. £5,000	10·0	5·1	678
£5,000 .. .. £10,000	5·1	7·5	914
£10,000 .. .. £50,000	2·7	18·0	983
£50,000 or more . . .	0·1	100·2	1,015

6. Explain the advantages of a logarithmic scale, using as an illustration data derived from some of those in question 5.

7. Use the method of moving averages in order to obtain the seasonal variation in the following series :—

Cost of Living Index Number

Month.	1931.	1932.	1933.	1934.
1	153	147	142	142
2	152	147	141	141
3	150	146	139	140
4	147	144	137	139
5	147	143	136	137
6	145	142	136	138
7	147	143	138	141
8	145	141	139	142
9	145	141	141	143
10	145	143	141	143
11	146	143	143	144
12	148	143	143	144

8. Explain the advantages of using weighted averages in the computation of index numbers. How far is this possible in the case of wholesale price indices?

9. Explain the need for exact definition in statistics. Illustrate with reference to statistics of trade and transport and unemployment.

10. What are the advantages and disadvantages of the *sample* method of statistical inquiry.

THE BUILDING SOCIETIES INSTITUTE

SECOND EXAMINATION

ELEMENTS OF STATISTICS

Thursday, 22nd June, 1939

(*Time allowed—two hours,* 4–6 *p.m.*)

Examiner : MR. G. L. SCHWARTZ, B.A., B.Sc.(Econ.)

(*Answer Question* 1 *and* THREE *other questions. All questions carry equal marks*)

1.

Persons in Household.	Number of Households.	Persons in Household.	Number of Households.
1	5	7	52
2	75	8	29
3	98	9	26
4	135	10	7
5	122	11	8
6	81	12	1

Treating the number of households as a frequency group, find the average, median and the upper and lower quartiles.

2. Plot a frequency curve for the data in Question 1 and use it to explain and measure skewness.

3. Select at random 20 numbers between 0 and 100 and then calculate the standard deviation and coefficient of dispersion for your group.

4. Discuss the need for exact definitions in statistics, illustrating with reference to population, housing accommodation or external trade statistics.

5. What averages are in common use in statistical treatment of data, and what are the special advantages and disadvantages of each ?

6. How is the spread of a statistical series about an average measured ? Arrange in order of variability of height policemen, the candidates in this examination room, guardsmen, a stage troupe of dancing girls, and housemaids. Give your reasons.

7. Construct a weighted index number of the cost of a budget in year *B* taking the cost in year *A* as 100, from the following data :—

	Year *A*.	Year *B*.	Weight.
	s. *d.*	*s.* *d.*	
Rent	9 0	12 0	20
Bread per 4 lb.	5	8	10
Butter per lb.	1 3	1 0	4
Meat per lb.	9	1 1½	8
Milk per quart	5	8	5
Coal per cwt.	1 6	1 10	3

8.

Income Class (Dollars).	Number of Persons in Class.	Amount of Total Income Received by Each Class (000 Dollars).
0 - 500	1,827	685
500 - 1,000	12.530	9,818
1,000 - 1,500	12,498	15.926
1,500 - 2,000	5,222	8,918
2,000 - 3,000	3,065	7,314
3,000 - 5,000	1.383	5,174
5,000 - 10,000	587	3,937
10,000 - 25,000	192	2.808

Draw two circular pictograms, one showing the number of persons in each class, and one showing the total income received by each class.

9.

Class.	Number of Cases.	
	Year 1890.	Year 1900.
20 +	1	2
15 - 20	3	2
12 - 15	14	6
10 - 12	15	6
9 - 10	4	3
8 - 9	5	17
7 - 8	3	14
6 - 7	4	4
5 - 6	4	1
4 - 5	2	3

Reduce the distributions to a common basis by expressing each in percentage terms and represent them by cumulative diagrams drawn on the same graph.

NATIONAL ASSOCIATION OF LOCAL GOVERNMENT OFFICERS

INTERMEDIATE EXAMINATION

January, 1936

ELEMENTS OF STATISTICS

(*Time allowed—2½ hours*)

(FOUR *questions—and only four—should be answered*)

1. Estimated speeds, immediately before the accidents, of mechanically-propelled vehicles involved in a number of fatal road accidents in the early part of 1935 :—

Estimated Speed.	Number of Vehicles.
Not exceeding 10 m.p.h.	190
Over 10 but not over 20 m.p.h.	509
,, 20 ,, ,, 30 ,,	321
,, 30 ,, ,, 40 ,,	86
,, 40 ,, ,, 50 ,,	23
,, 50 m.p.h.	5
Vehicle stationary	31
Information not available	361
	1,526

Calculate (i) the percentages of vehicles moving at different speeds; (ii) the average speed of moving vehicles; (iii) the modal speed of moving vehicles.

2. Using the data in Question 1 draw a frequency diagram representing the speeds of vehicles involved in fatal road accidents.

3. Using the data in Question 1 draw a cumulative diagram from which the median speed of moving vehicles involved in fatal road accidents can be read off. Write down the median speed.

4. Assuming that the estimates of the speeds of vehicles tabulated in Question 1 are correct, can it be inferred that fewer fatal accidents would probably occur if vehicles moved in future at (i) higher, or (ii) lower, speeds than in the early part of 1935? Give reasons for your conclusion.

5. Present the following information, in its present or a more convenient form, in a table or tables suitable for publication :—

According to the Census of Production, 1930, the total value of boots, shoes and slippers made in the United Kingdom in that year was £41,473,000, a decrease of 12·6 per cent. as compared with 1924. The total number of dozen pairs made in 1924 was 9,906 thousand, and in 1930 this number had decreased by 212 thousand. In 1930 there were 2,121,000 dozen pairs of men's leather boots and shoes (value £14,459,000), 3,146,000 dozen pairs of women's (value £16,274,000), 2,361,000 dozen pairs of children's (value £6,080,000), and 293,000 dozen pairs of leather shoes (worth £1,654,000) not separately distinguished as men's, women's or children's. There were in addition 1,773 thousand dozen pairs of footwear of other materials, which were worth £3,006,000. In 1924 the values of men's, women's and children's leather boots and shoes were £16,099,000, £18,135,000, and £7,286,000 respectively. The total number of pairs was, as already stated, 9,906,000 dozens, of which 73·2 per cent. were of leather (21·4 per cent. men's, 27·7 per cent. women's, and 24·1 per cent. children's). The material of the remaining 26·8 per cent. was not stated and may have been either leather or some other material; their value was £5,920,000.

6. Calculate, from the data in the following table, the average percentage change between 1924 and 1930 in the prices of hats and caps :—

Censuses of Production (United Kingdom)

	1930.		1924.	
	Quantity (Thousand dozens).	Value (£000).	Quantity (Thousand dozens).	Value (£000).
Hats and caps :				
of straw	957	2,305	833	2,434
of wool felt	1,577	2,801	1,291	2,622
of fur felt	826	3,352	524	3,100
of cloth	1,406	1,390	1,654	1,918
of other materials	260	558	248	715
	5,026	10,406	4,550	10,789

7. Discuss briefly the nature and use of three (and only three) of the following :—

(i) index-numbers; (ii) moving averages; (iii) quartiles; (iv) the census of population; (v) standardised death rates.

NATIONAL ASSOCIATION OF LOCAL GOVERNMENT OFFICERS

INTERMEDIATE EXAMINATION

May, 1939

ELEMENTS OF STATISTICS

(*Time allowed—2½ hours*)

(*Candidates should answer* FOUR *questions*)

1. Explain how the Net Reproduction Rate is obtained. Show the significance of the fact that in 1931 the net reproduction rate for England and Wales was 0·81.

2. What are the principal Local Government matters in respect of which statistics are regularly collected and published? Where can this information be found? Indicate what other matters (if any) could, in your opinion, usefully be treated statistically.

3. Put the following information in tabular form: On the 12th December, 1938, there were in Great Britain 1,804,218 insured persons aged 16–64, unemployed, of whom 1,380,304 were Men (aged 18 and over), 371,550 were Women, 29,028 were Boys (under 18), and 23,336 were Girls. The total numbers on the registers of the Employment Exchanges were: Men, 1,384,922, Women, 356,580, Boys, 45,549 and Girls 44,321. Explain the differences between the two sets of figures.

4. State concisely the bases of good classification of statistical data. Consider how far the classification of British trade statistics satisfies theoretical requirements.

5. The estimated age composition of the population of England and Wales in 1937 was, in Millions :—

	0–	10–	20–	30–	40–	50–	60–	70–	80–
M.	2·91	3·30	3·29	3·15	2·49	2·17	1·57	0·68	0·14
F.	2·83	3·24	3·40	3·36	2·95	2·55	1·83	0·92	0·24

Draw the population pyramid for 1937, and comment on the position it reveals, taking into account the average (crude) birth and death rates for the periods given :—

	1921–5.	1926–30.	1931–5.
Birth rate . .	19·9	16·7	15·0
Death rate . .	12·1	12·1	12·0

6. Compare graphically the expenditures of Local Authorities in England and Wales during the stated period under the three headings given, so as to bring out clearly the different rates of growth in expenditure :—

Education :	1920–1.	1921–2.	1922–3.	1923–4.	1924–5.
(1) Elementary . .	59,855	62,116	59,729	57,917	58,720
(2) Higher . .	14,030	15,688	15,061	14,382	15,153
Housing and Town Planning . .	4,315	10,214	14,631	16,327	18,083
Education :	**1925–6.**	**1926–7.**	**1927–8.**	**1928–9.**	
(1) Elementary . .	59,631	59,896	59,832	63,017	£000
(2) Higher. . .	15,876	16,144	17,163	18,690	£000
Housing and Town Planning . .	21,282	26,167	32,186	36,615	£000

7. You have been instructed to find the "general level of working-class incomes" in the district served by your authority, to be used as a guide to policy in such matters as the determination of the rents that should be charged in a proposed housing estate. How would you work out a single expression for working-class income given that :—

(i) A variety of industrial occupations is to be found in your area;

(ii) The employees are all paid weekly time-rates;

(iii) You have all requisite date as to employment for the period over which your investigation extends.

8. According to the Census of 1931, the following was the age-distribution of clerks employed in Local Government :—

Age . .	14 & 15	16 & 17	18–20	20–24	25–29	30–34
No. . .	383	2,732	4,894	5,501	4,256	3,857
Age . .	35–44	45–54	55–59	60–64	65–69	
No. . .	5,973	4,051	1,337	928	363	
Age . .	70 and over					
No. . .	217					

All ages 34,492.

Calculate the Average, Median and Quartile ages of this group, and give the Relative Mean Variation from the average.

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